Principles of Communications
ECS 332

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6. Sampling, Reconstruction, and Pulse Modulation

Office Hours:

BKD, 4th floor of Sirindhralai building

Monday	9:30-10:30
Monday	14:00-16:00
Thursday	16:00-17:00

Principles of Communications
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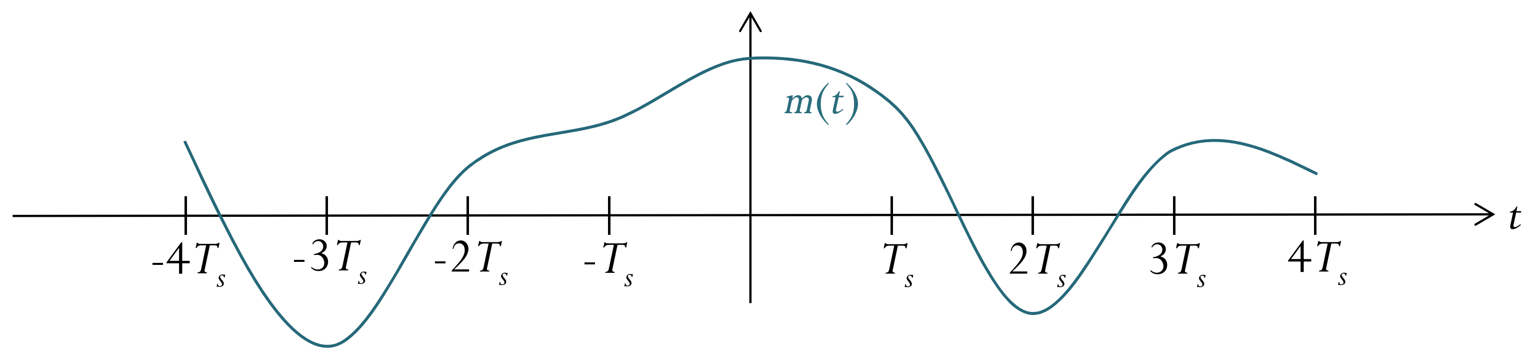
6.1 Sampling

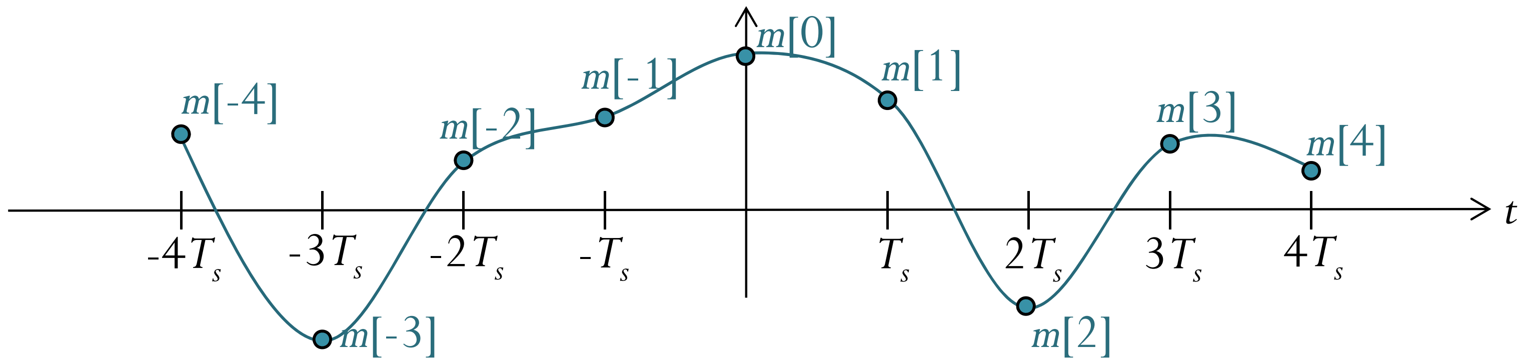
Sampling

- Start with a continuous-time (analog) signal.



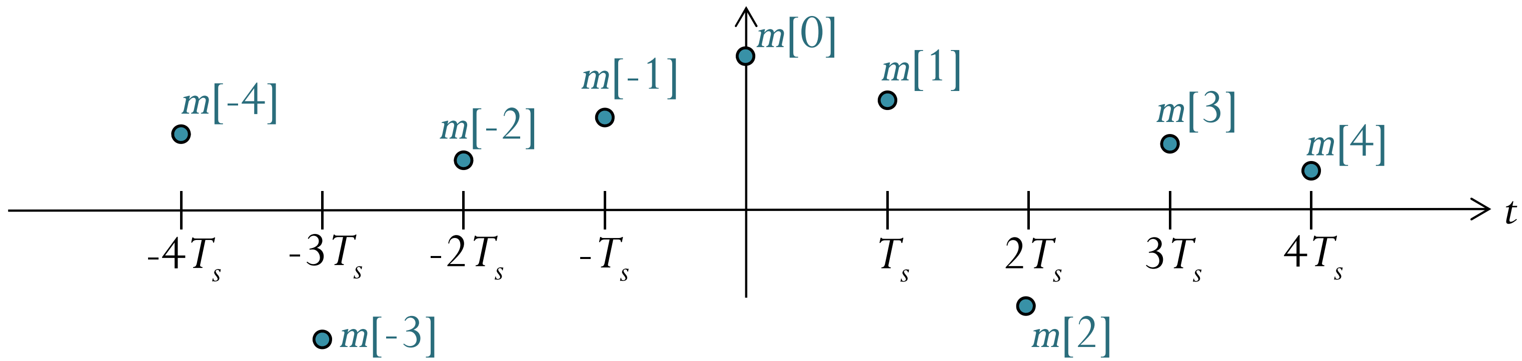
Sampling

- Record the value every T_s seconds.



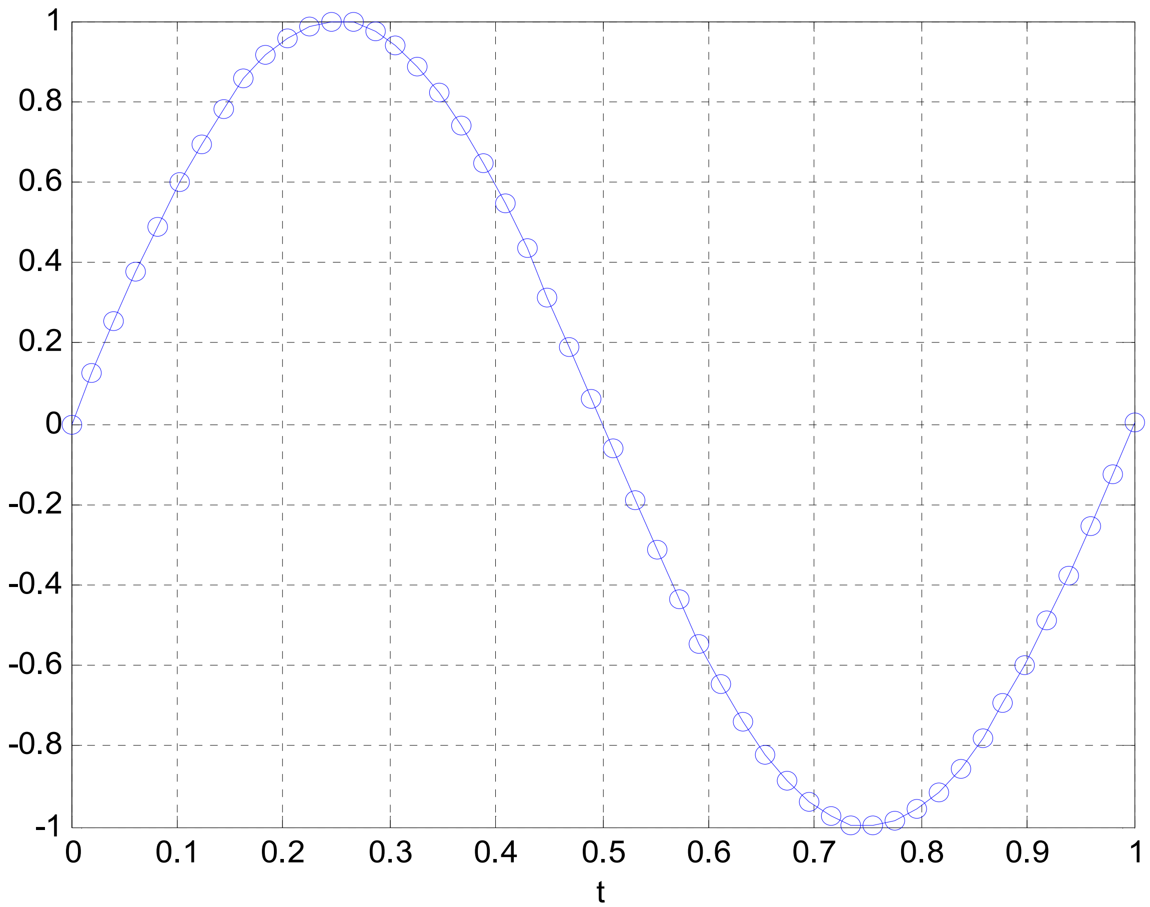
Sampling

- Get a sequence of samples (numbers).



Example: $\sin(100\pi t)$ (1/4)

This is the plot of $\sin(100\pi t)$. What's wrong with it?



[AliasingSin_2.m]

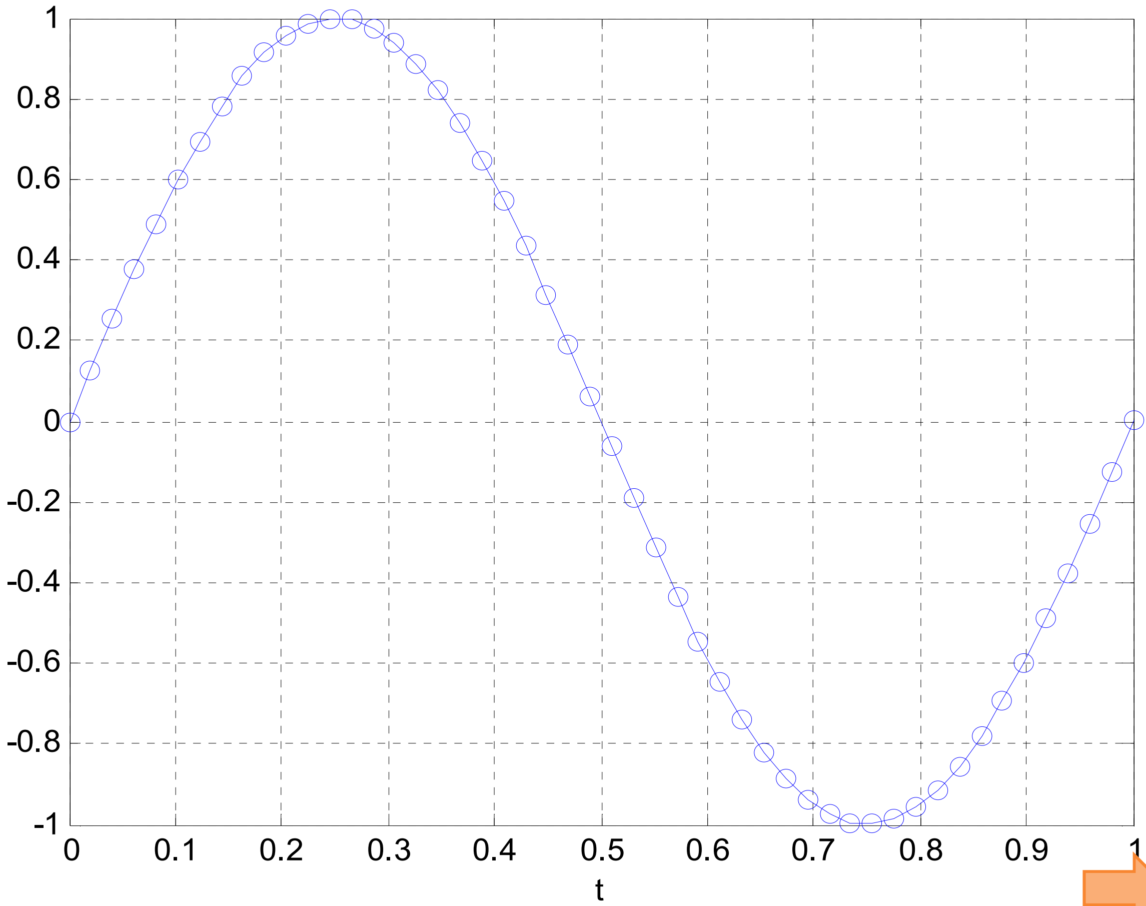
Example: sin(100πt) (2/4)

Signal of the form $\sin(2\pi f_0 t)$ have frequency $f = f_0$ Hz.

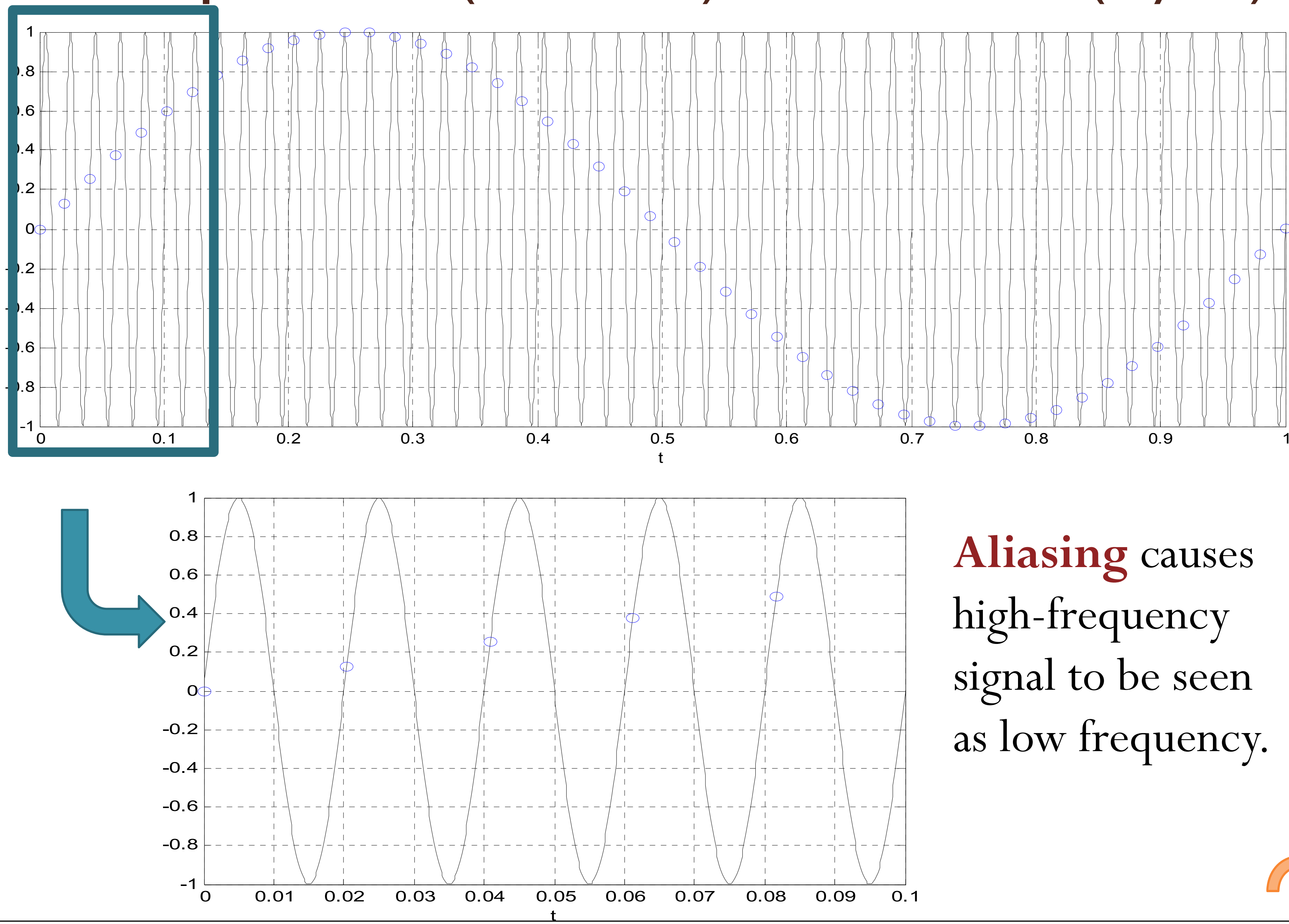
So, the frequency of $\sin(100\pi t)$ is 50 Hz.

From time 0 to 1, it should have completed 50 cycles. However, our plot has only one cycle.

It looks more like the plot of $\sin(2\pi t)$



Example: sin(100πt) (3/4)



Aliasing causes high-frequency signal to be seen as low frequency.

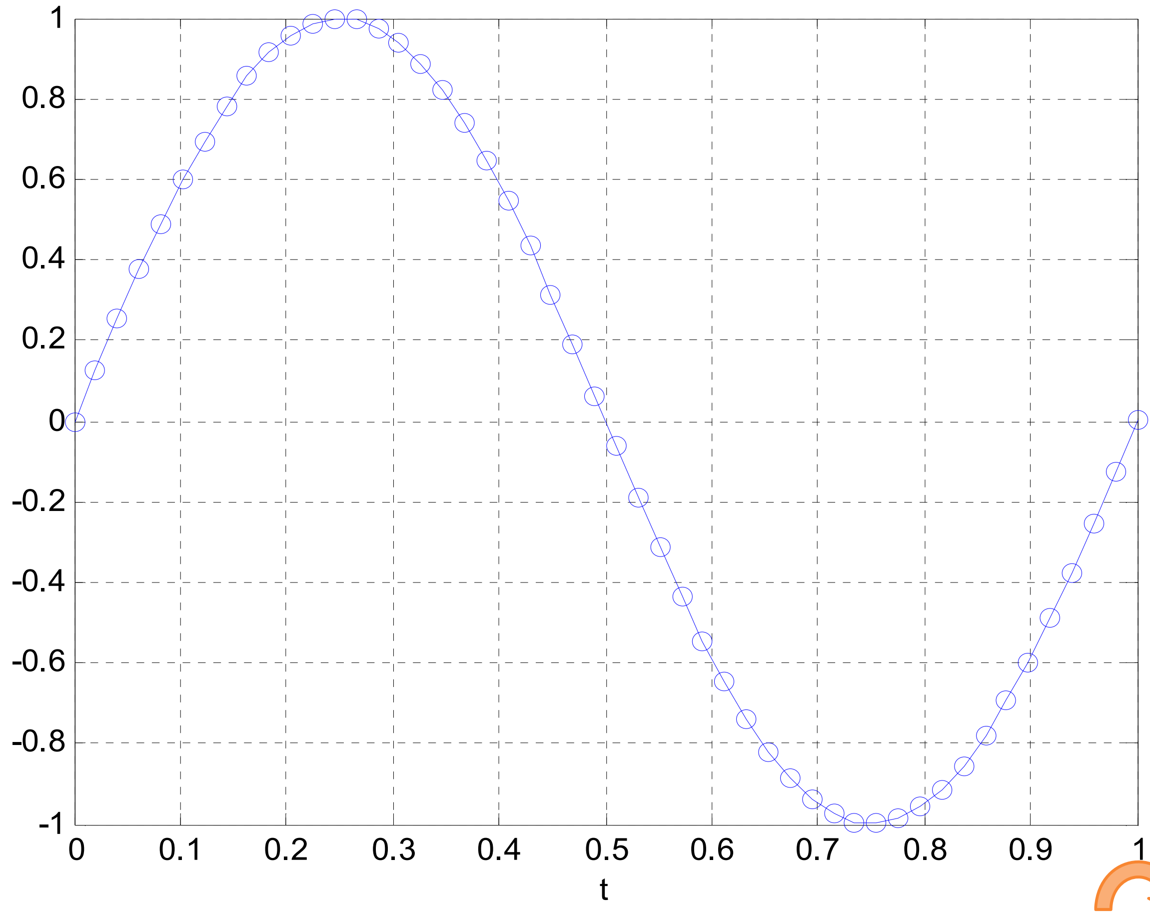
Example: sin(100πt) (4/4)

Signal of the form $\sin\left(2\pi f_0 t\right)$ have frequency $f = f_0$ Hz.

So, the frequency of $\sin\left(100\pi t\right)$ is 50 Hz.

We need to sample at least 100 times per time unit.

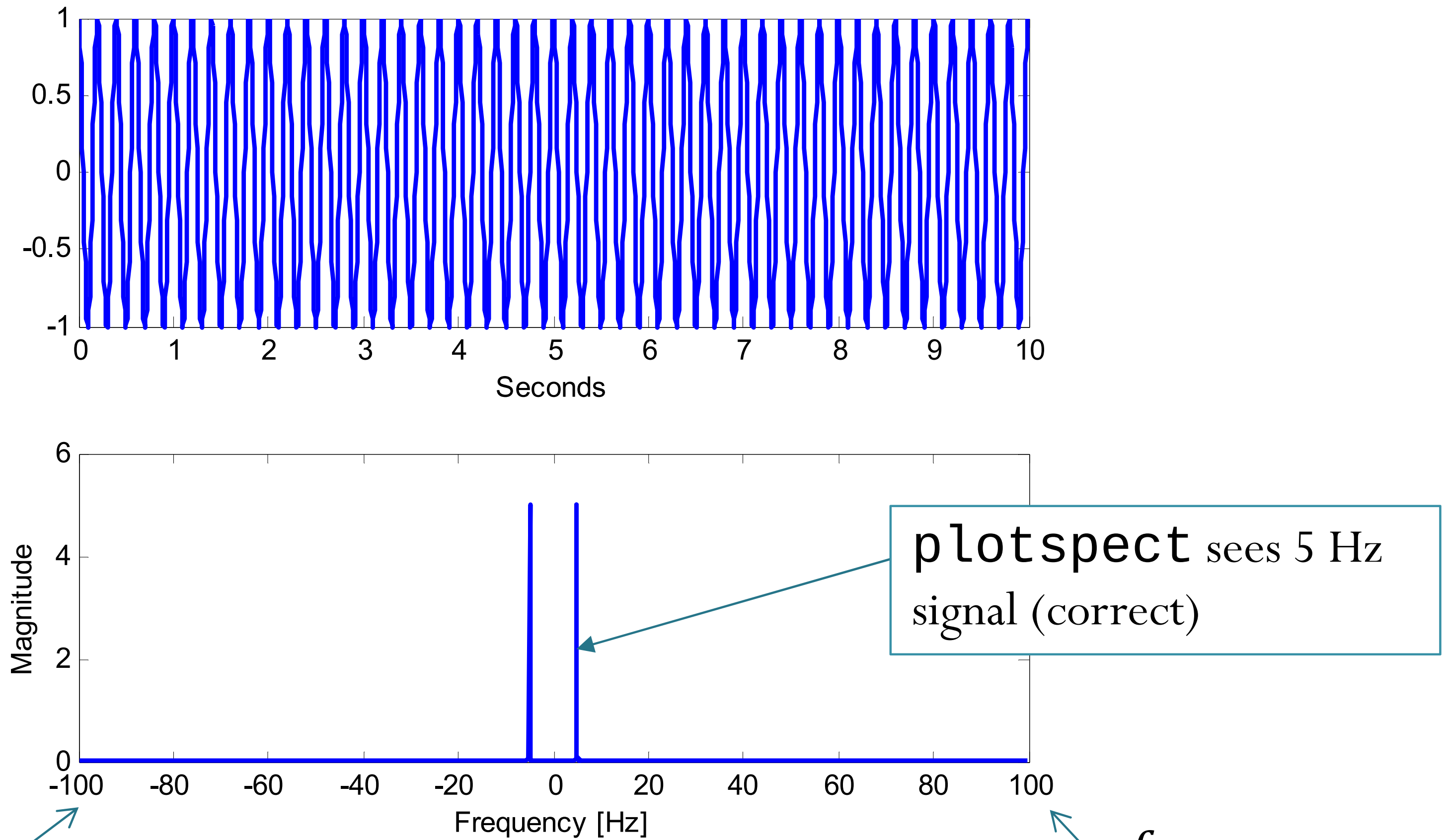
Here, the number of sample per time unit is 49, which is too small to avoid aliasing.



Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(5)t)$

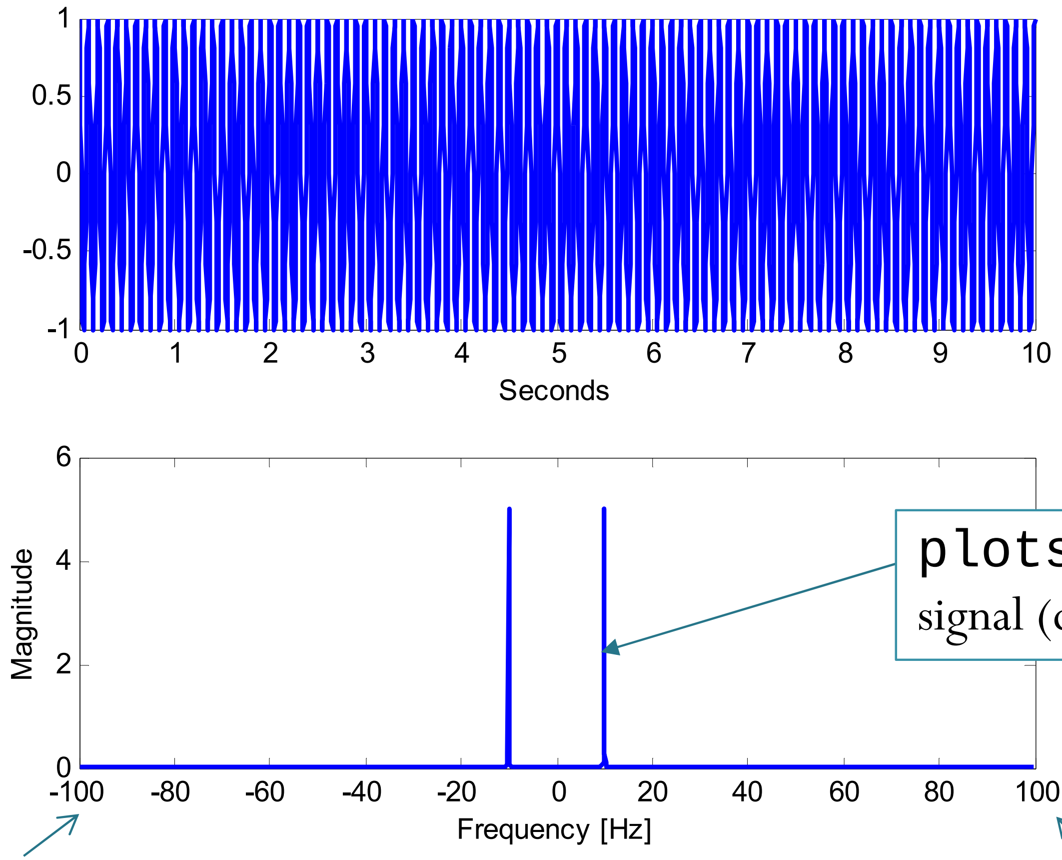


$-\frac{f_s}{2}$ $\frac{f_s}{2}$

Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(10)t)$



`plotspect` sees 10 Hz signal (correct)

$-\frac{f_s}{2}$ $\frac{f_s}{2}$

Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(50)t)$

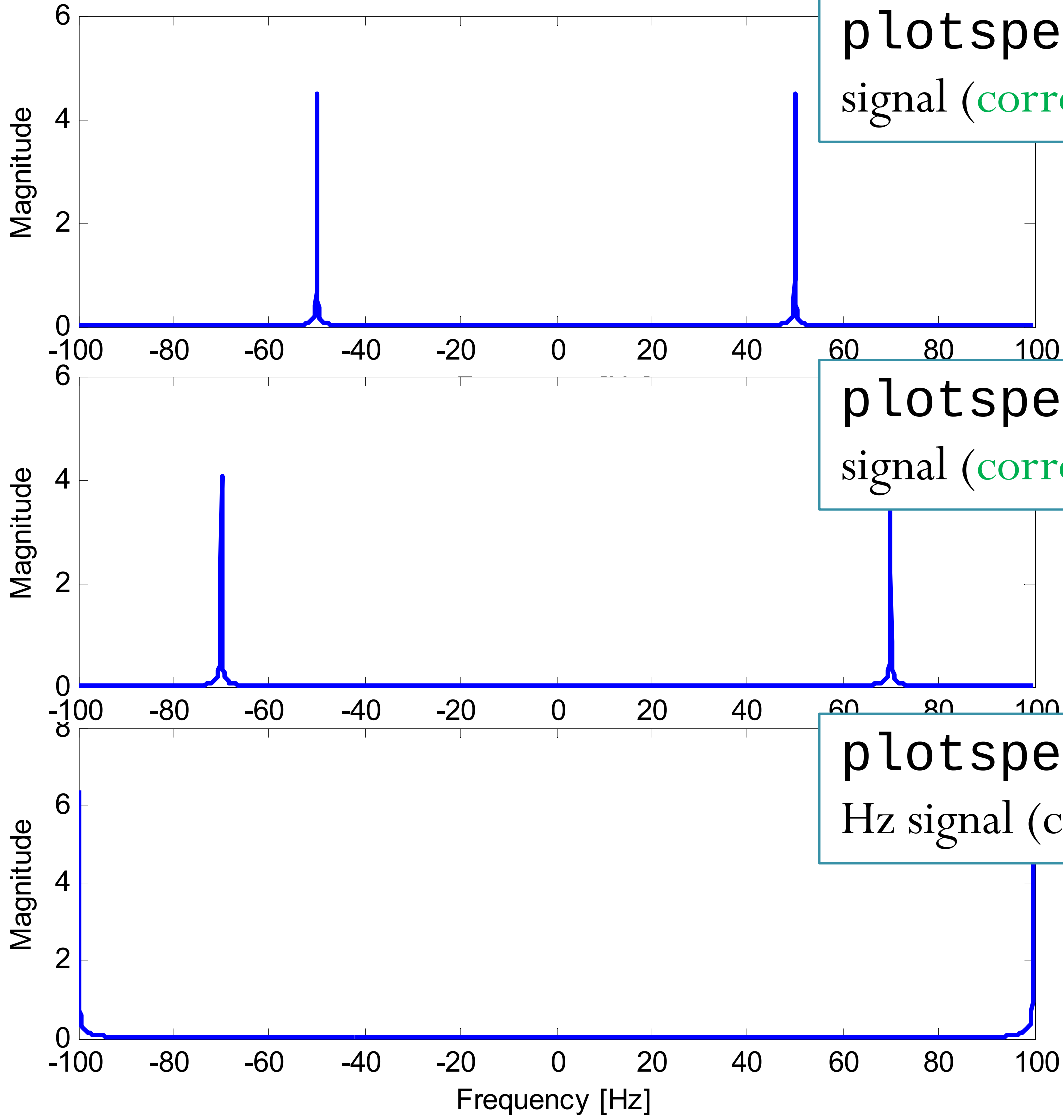
`plotspect` sees 50 Hz signal (correct)

$\cos(2\pi(70)t)$

`plotspect` sees 70 Hz signal (correct)

$\cos(2\pi(100)t)$

`plotspect` sees 100? Hz signal (correct?)



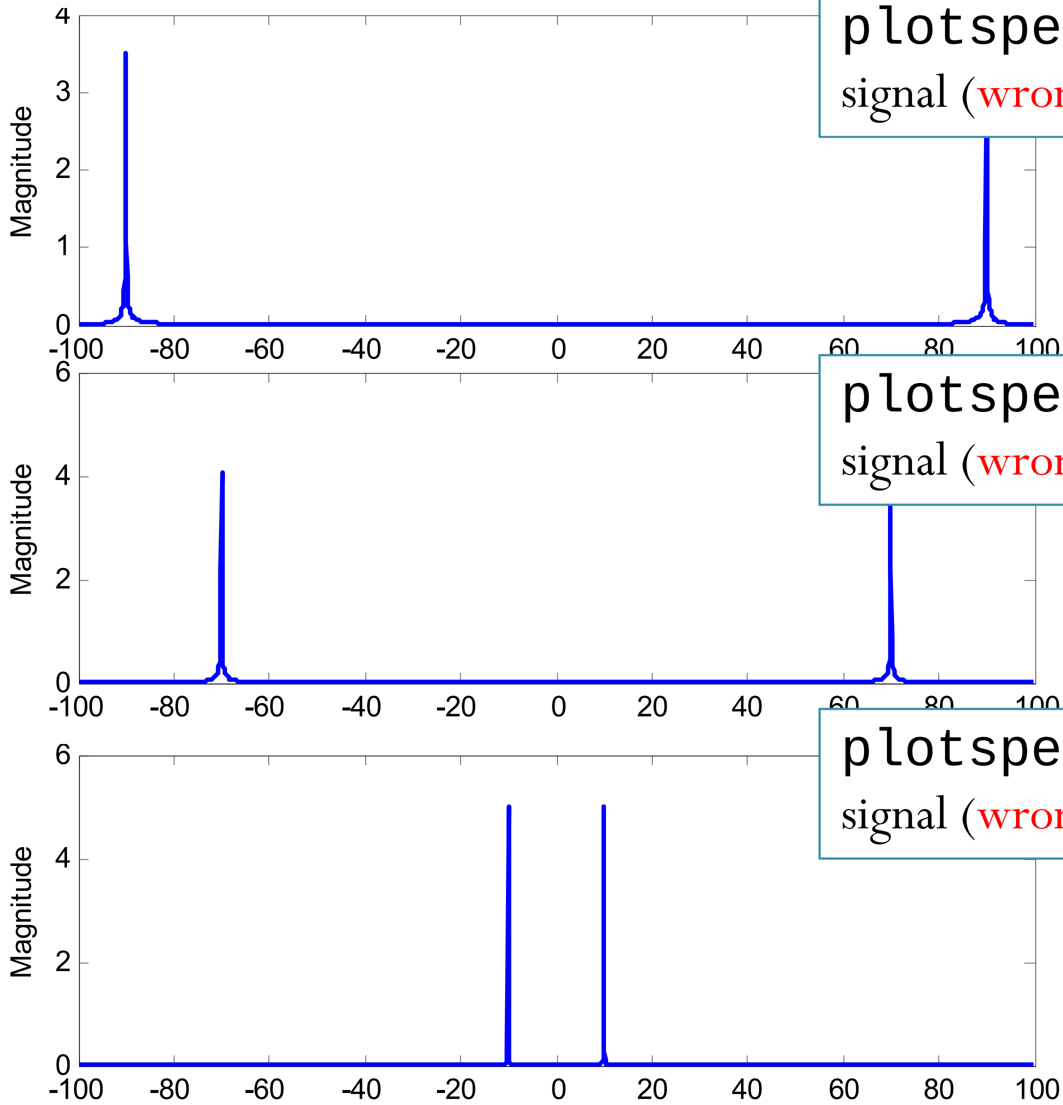
Using plotspect.m to study aliasing

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(110)t)$

$\cos(2\pi(130)t)$

$\cos(2\pi(190)t)$



`plotspect` sees 90 Hz signal (wrong!)

`plotspect` sees 70 Hz signal (wrong!)

`plotspect` sees 10 Hz signal (wrong!)

Using plotspect.m to study aliasing

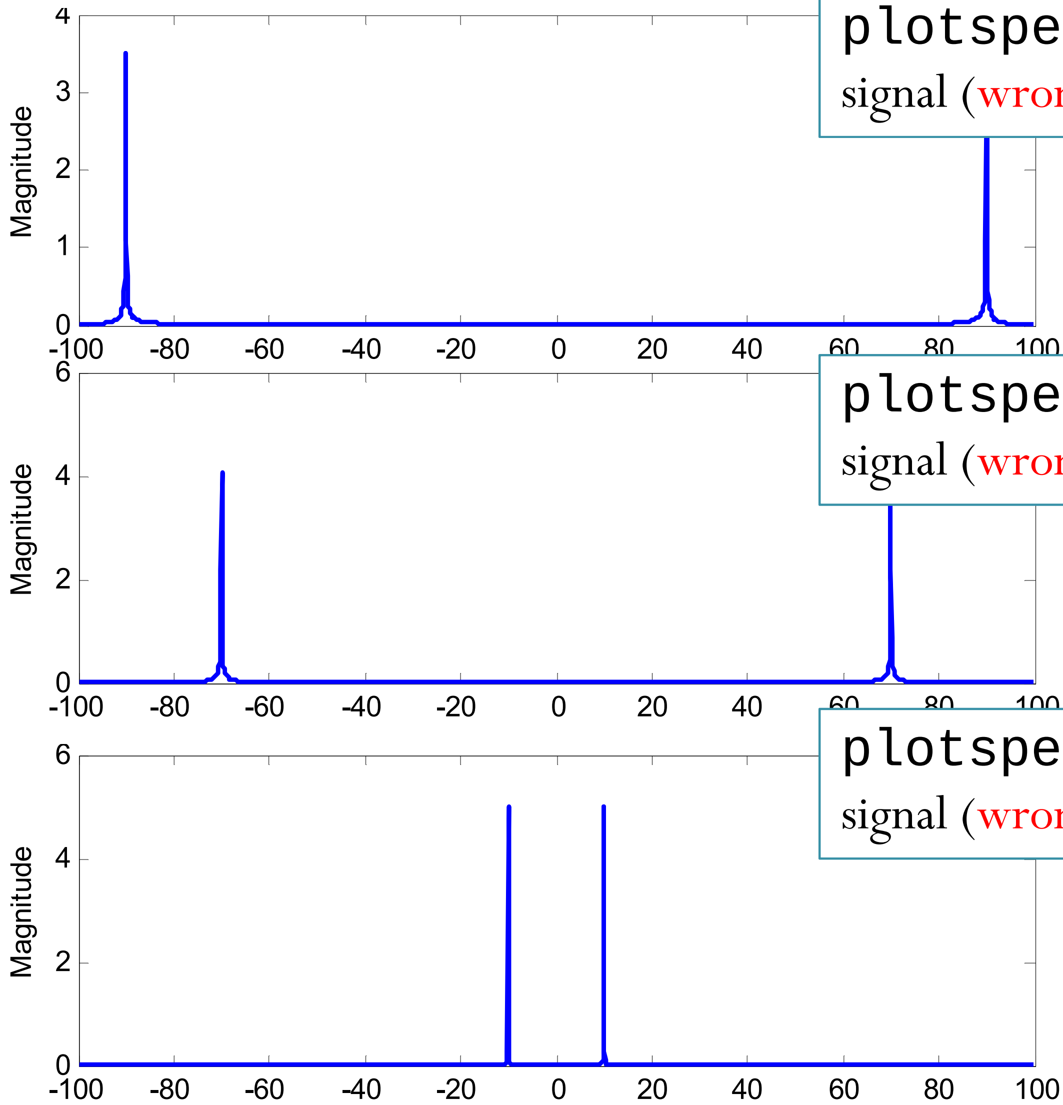
- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(110)t)$

$\cos(2\pi(130)t)$

$\cos(2\pi(190)t)$

This behavior is commonly referred to as **folding**.



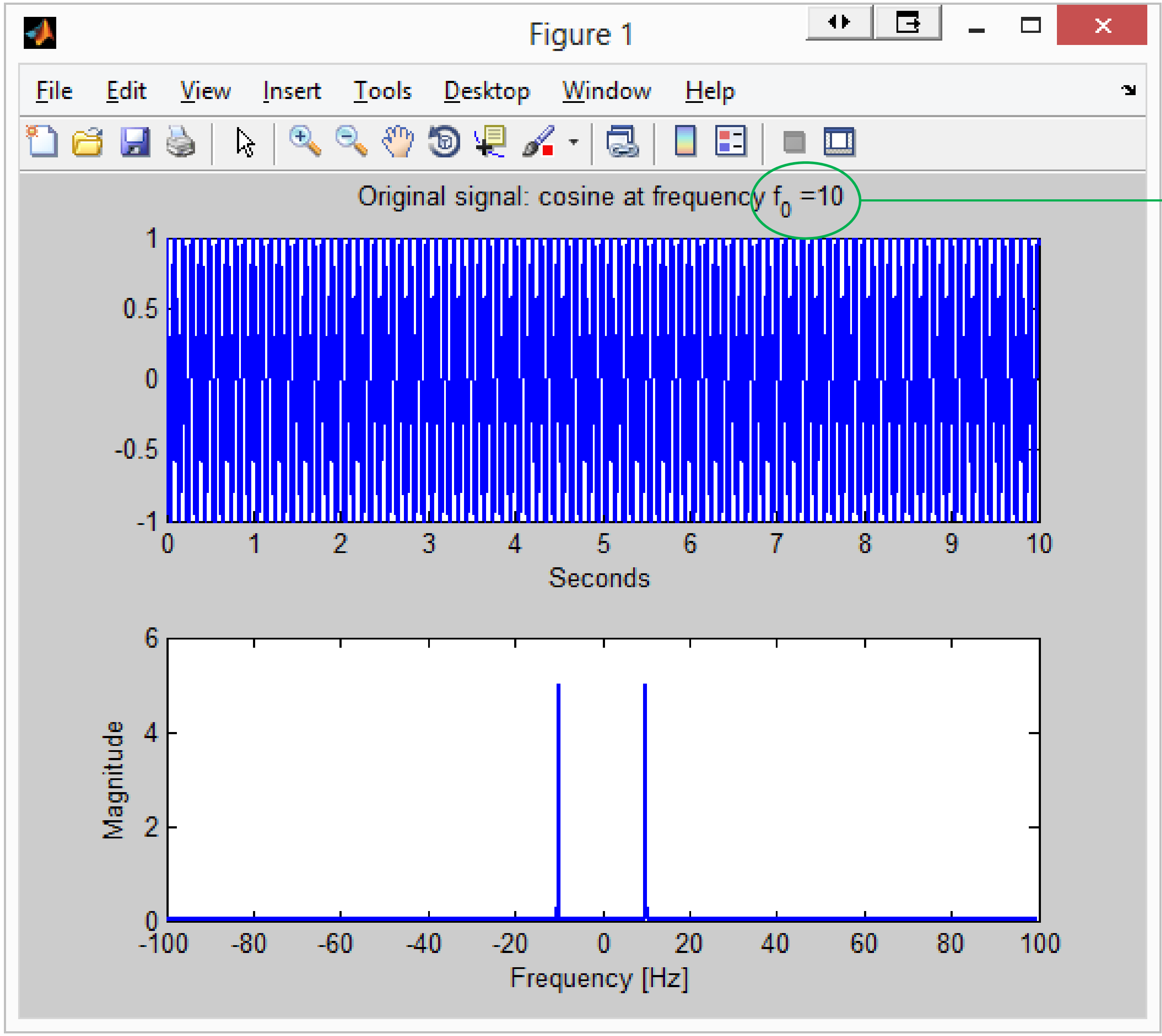
`plotspect` sees 90 Hz signal (wrong!)

`plotspect` sees 70 Hz signal (wrong!)

`plotspect` sees 10 Hz signal (wrong!)

MATLAB Demo

f_s: Sampling frequency = 200 samples/sec

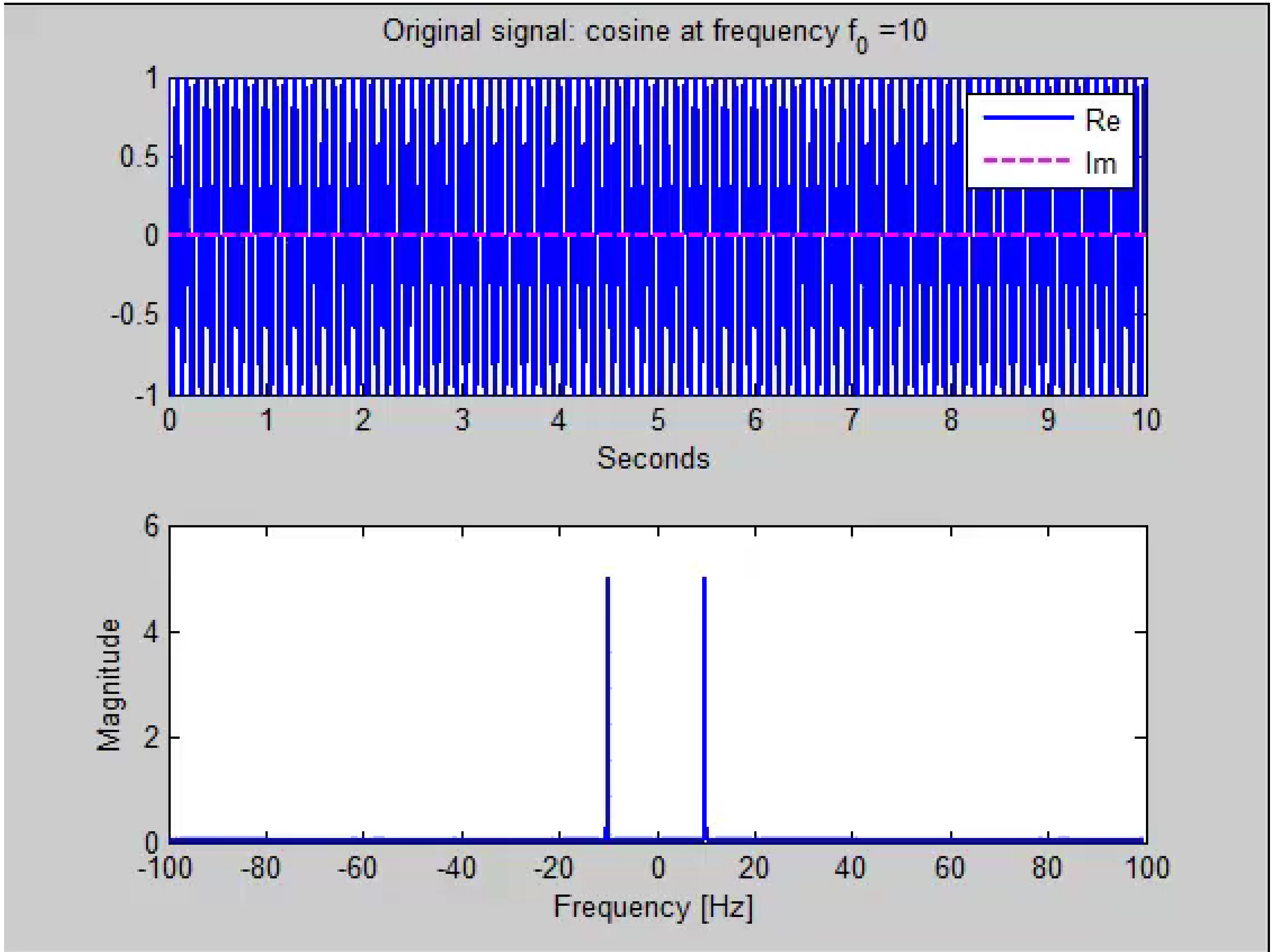


$\cos(2\pi(f_0)t)$

The frequency f_0 of the cosine is increased (in steps of 10) from 10 Hz to 300 Hz.

[aliasingCos.m]

MATLAB Demo



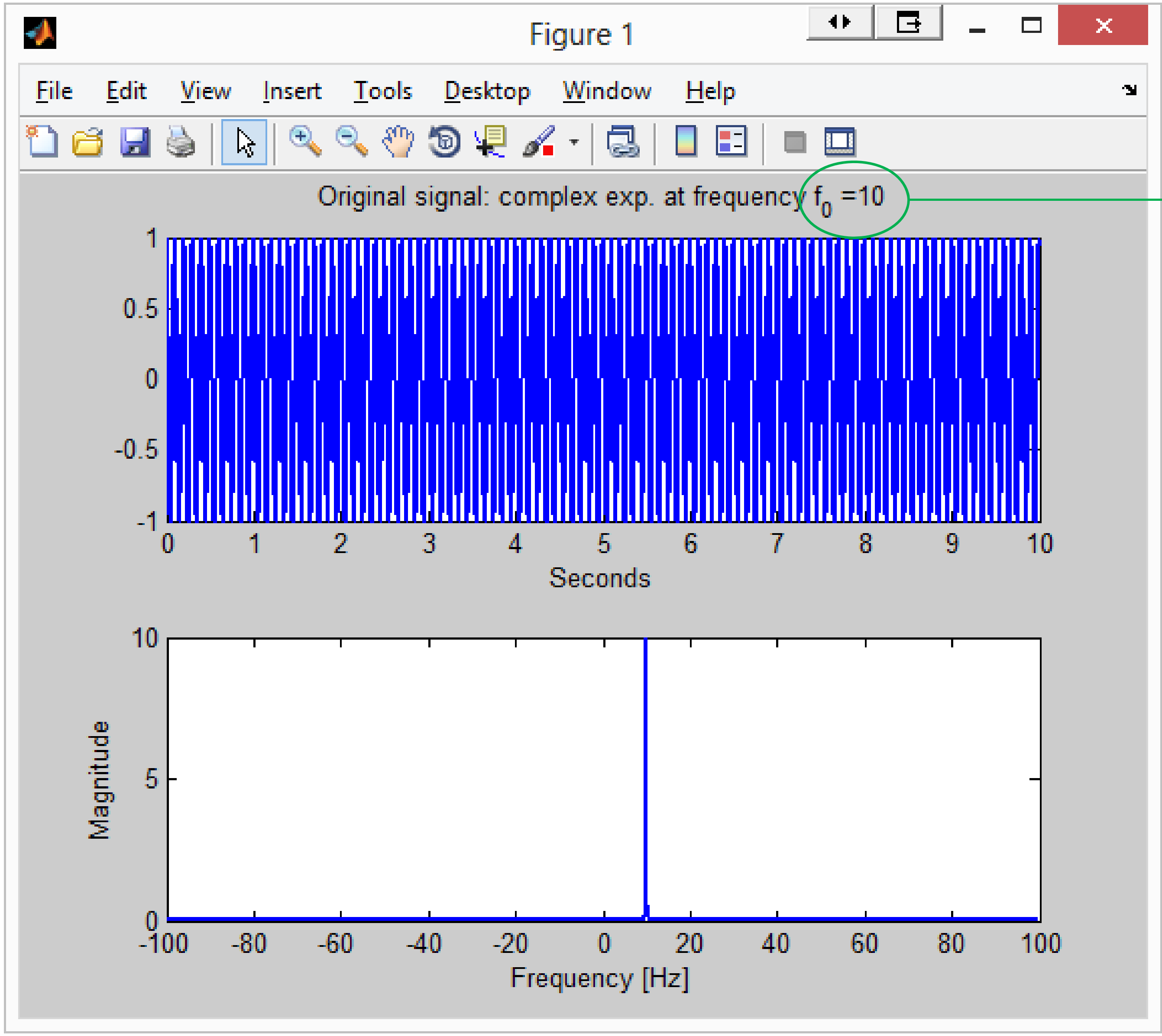
[aliasingCos.m]

Pac Man's Tunneling

Actually, I think we should call it **tunneling** (like in Pac Man).

MATLAB Demo

f_s: Sampling frequency = 200 samples/sec

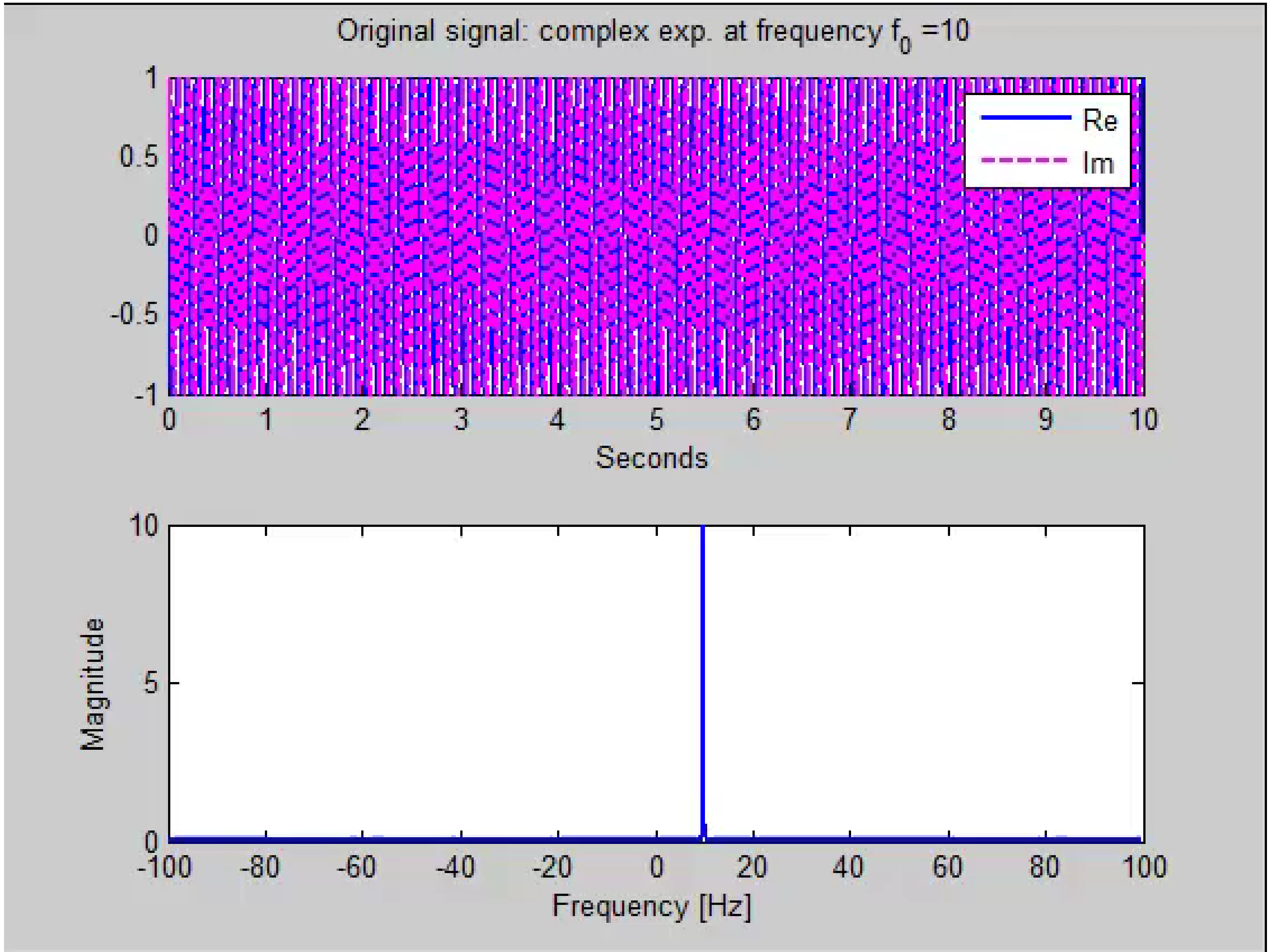


$$e^{j2\pi(f_0)t}$$

The frequency f_0 of the complex expo. signal is increased (in steps of 10) from 10 Hz to 300 Hz.

[aliasingExp.m]

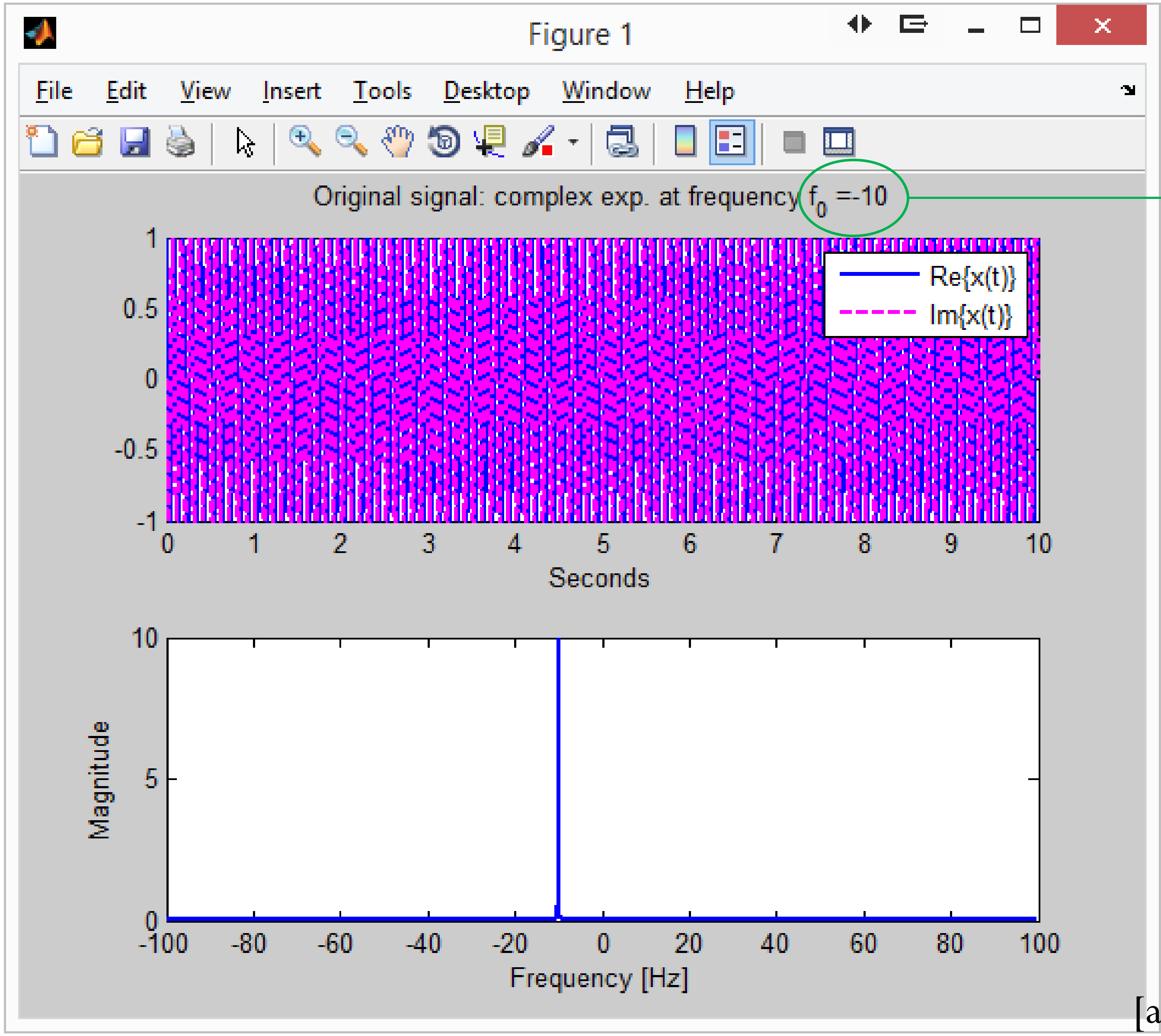
MATLAB Demo



[aliasingExp.m]

MATLAB Demo

f_s: Sampling frequency = 200 samples/sec

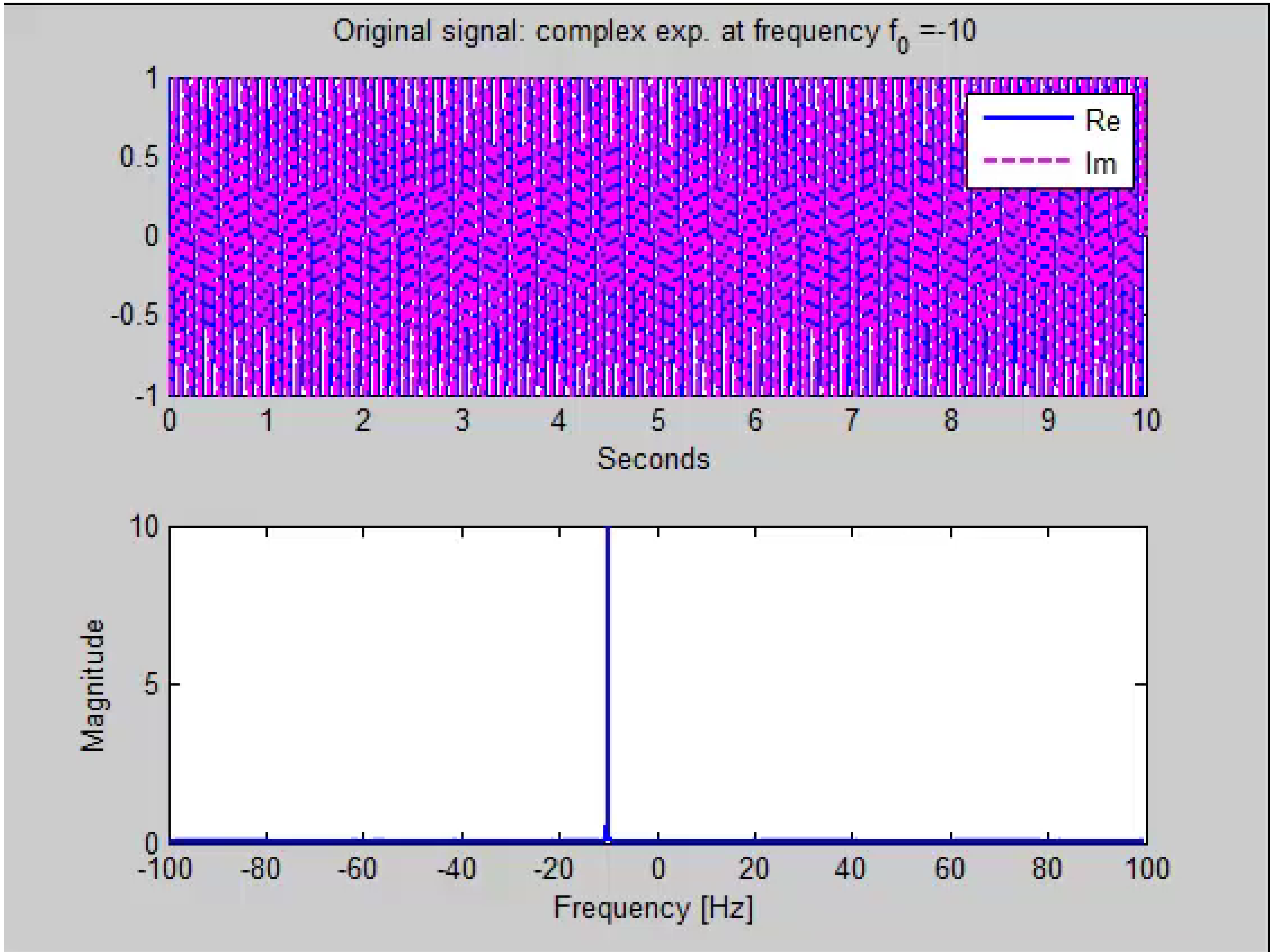


$$e^{j2\pi(f_0)t}$$

The frequency f_0 of the complex expo. signal is decreased (in steps of 10) from -10 Hz to -300 Hz.

[aliasingExpNegative.m]

MATLAB Demo

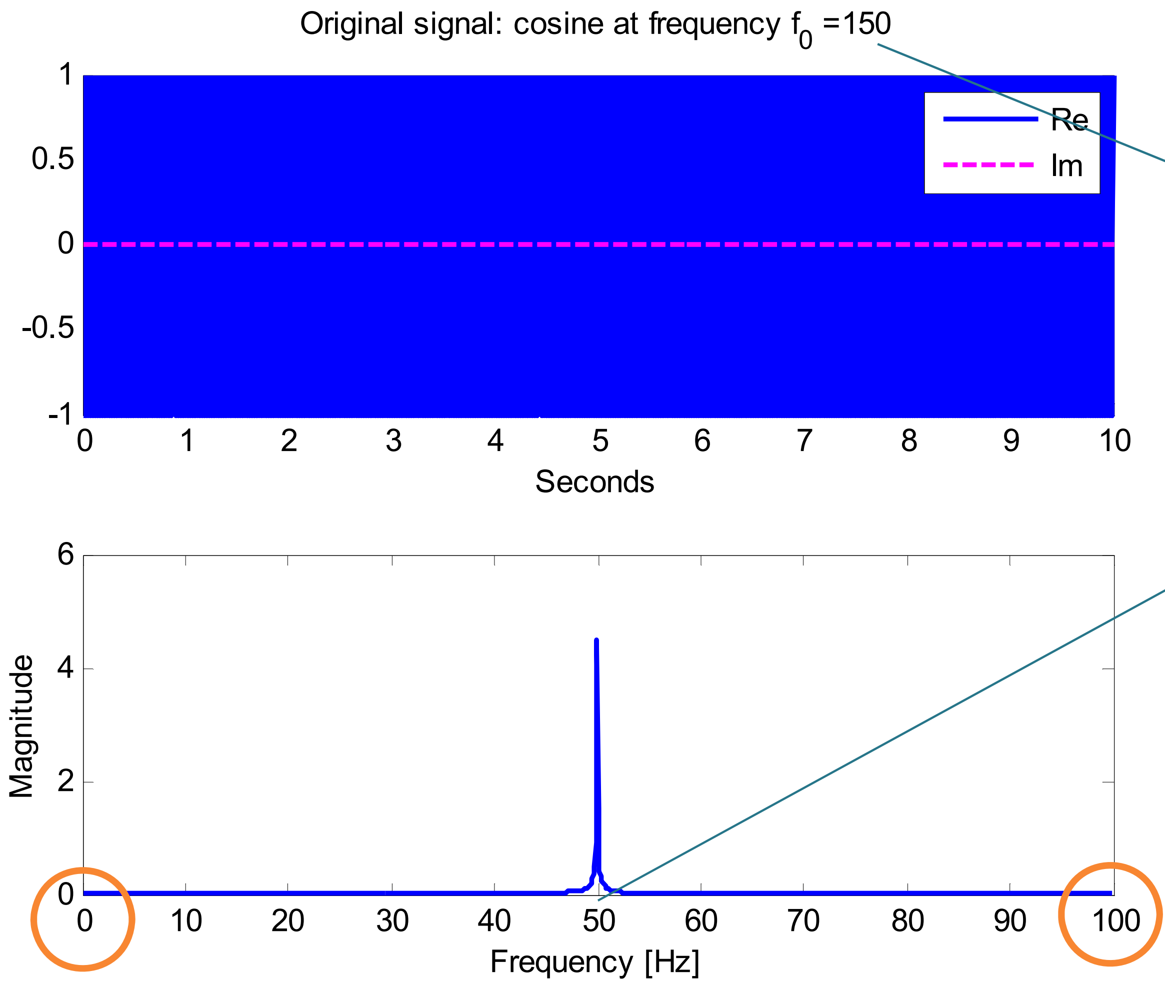


[aliasingExpNegative.m]

Conclusion

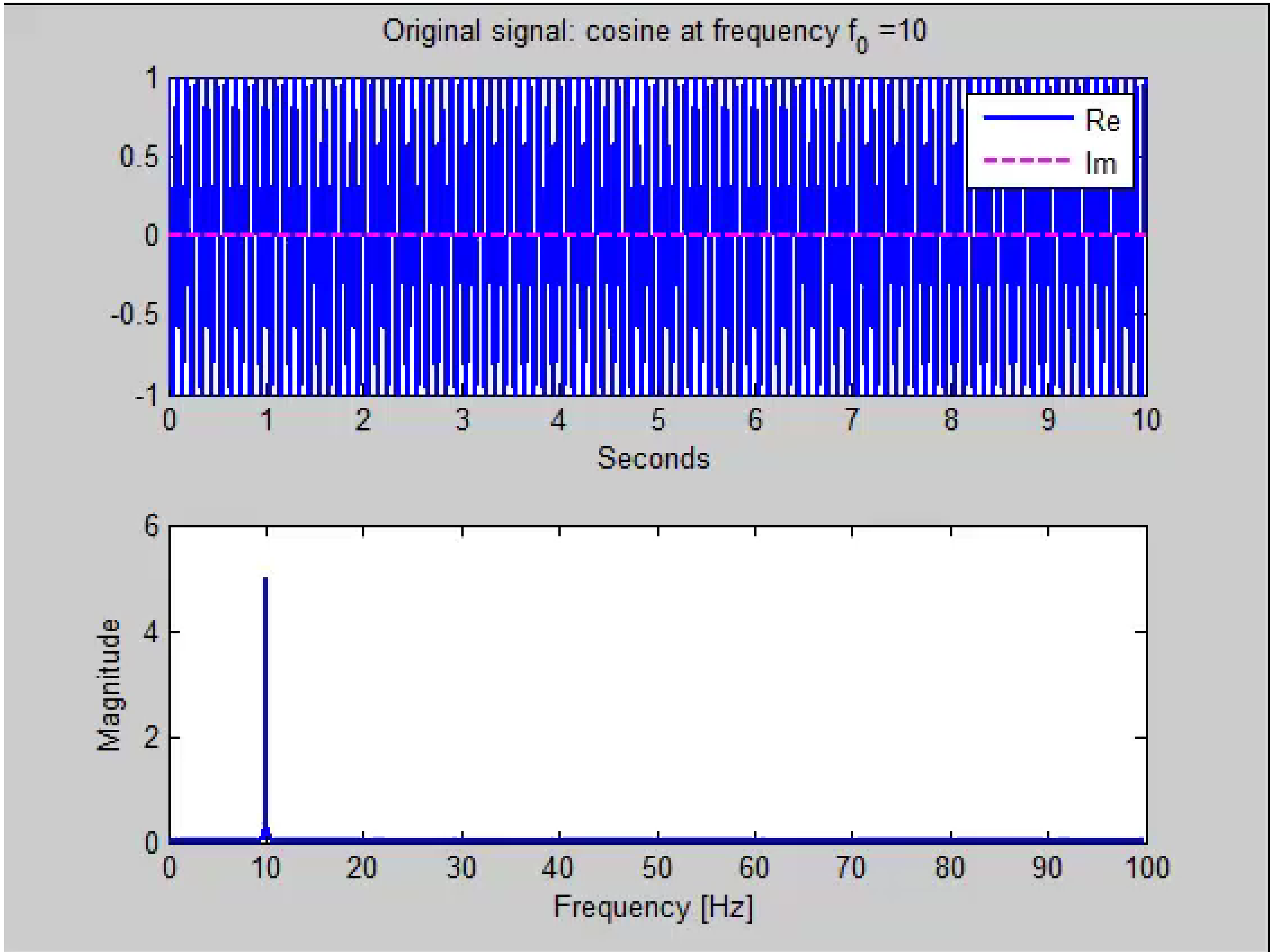
- The folding technique is useful for finding the perceived frequency of $\cos(2\pi(f_0)t)$.

Demo: [aliasingCos_folding]



When f_s = 200 [Sa/s], the cosine @ freq. 150 Hz will be perceived as a cosine @ freq. 50 Hz.

MATLAB Demo

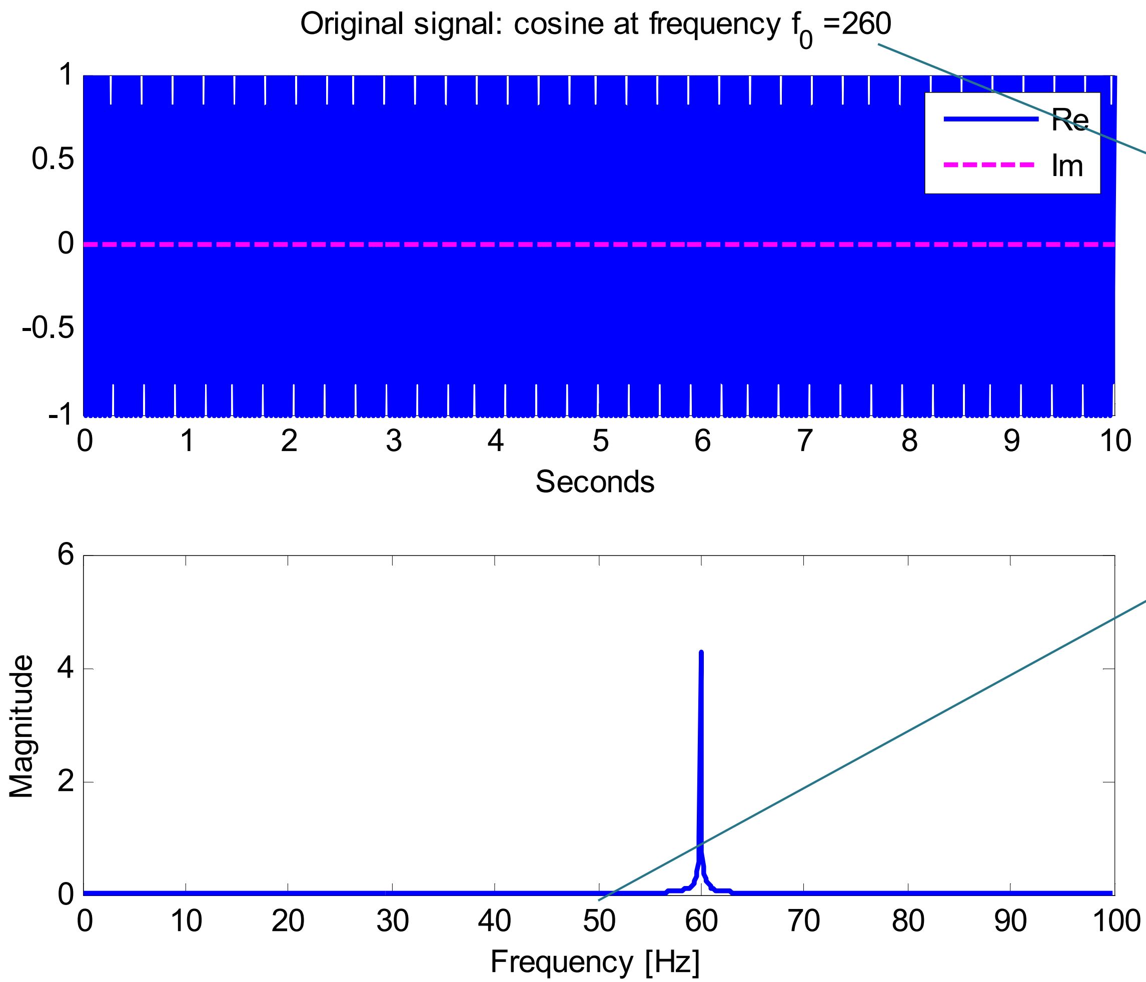


[aliasingCos_folding]

Conclusion

- The folding technique is useful for finding the perceived frequency of $\cos(2\pi(f_0)t)$.

Demo: [aliasingCos_folding]



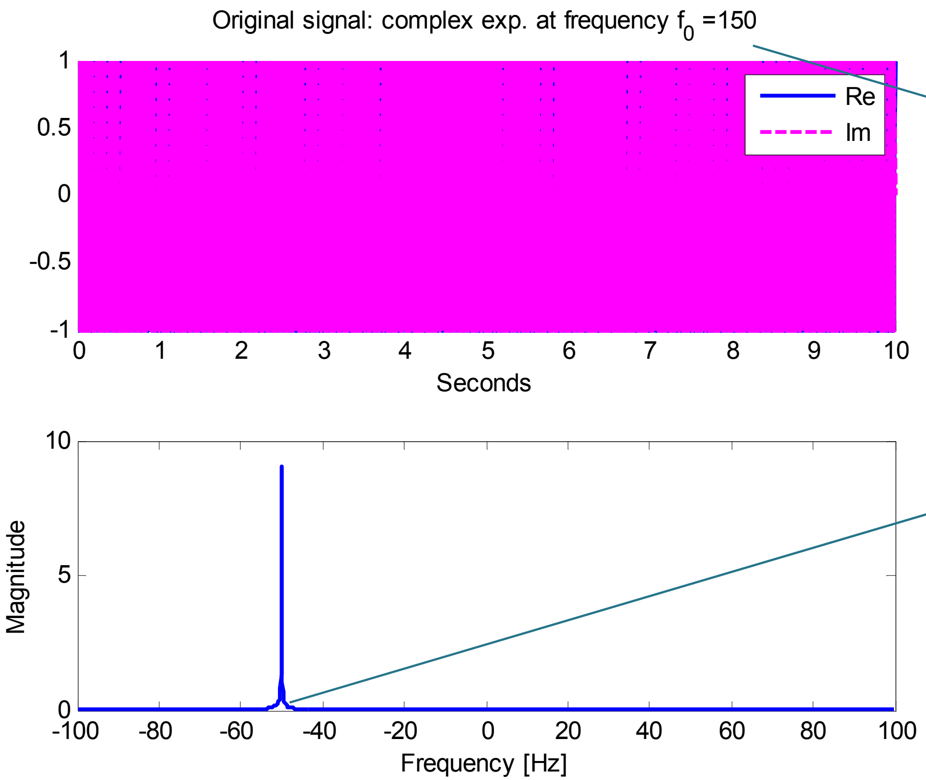
When $f_s = 200$ [Sa/s], the cosine @ freq. 260 Hz will be perceived as a cosine @ freq. 60 Hz.

Conclusions

- The folding technique is useful for finding the perceived frequency of $\cos(2\pi(f_0)t)$.
  - OK to look at the frequency only from 0 to $f_s/2$.
- When the signal does not have the "symmetry" between the positive and negative frequency parts,
  - for example, the complex exponential $e^{j2\pi(f_0)t}$
  - must look at the frequency from $-f_s/2$ to $f_s/2$.
- Actually, it is doing "tunneling".

Conclusions

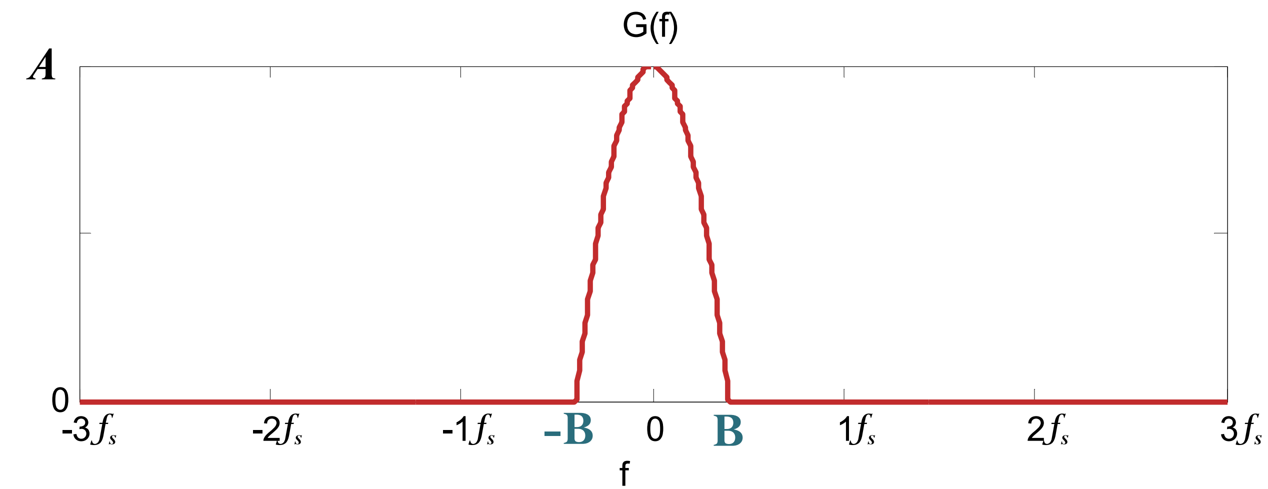
- When the signal does not have the "symmetry" between the positive and negative frequency parts,
  - for example, the complex exponential $e^{j2\pi(f_0)t}$
  - must look at the frequency from $-f_s/2$ to $f_s/2$.



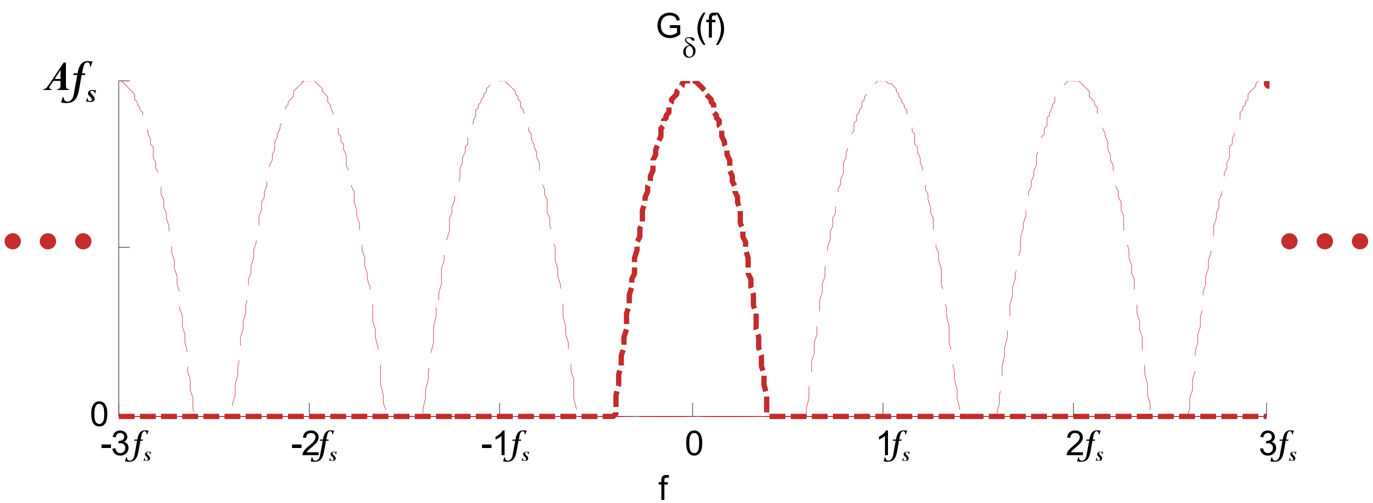
When $f_s = 200$ [Sa/s], the cosine @ freq. 150 Hz will be perceived as a cosine @ freq. -50 Hz.

Ideal Sampling

The Fourier transform of the original signal



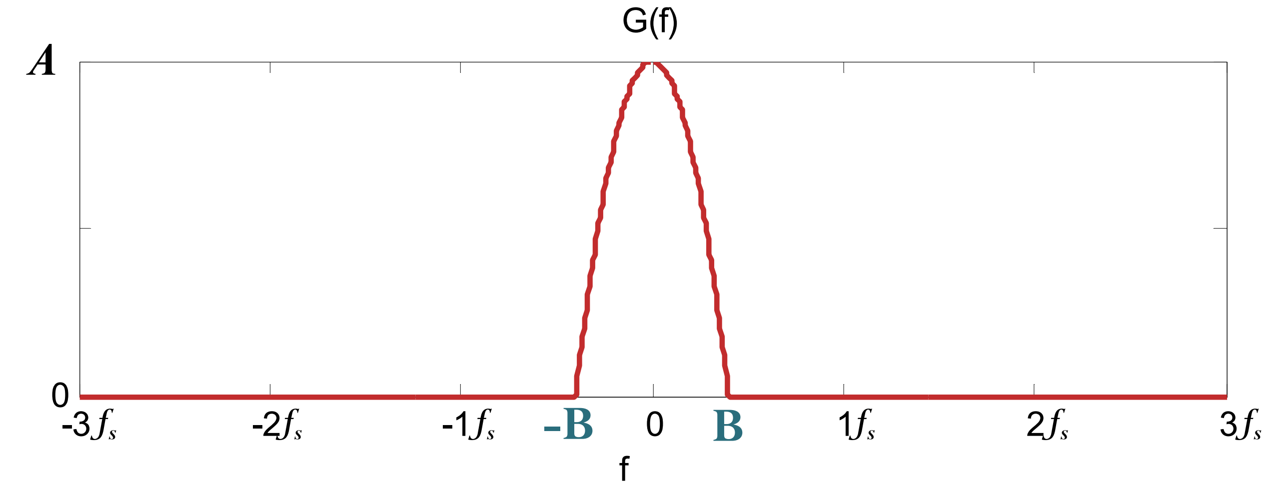
The Fourier transform of the (ideal) sampled signal



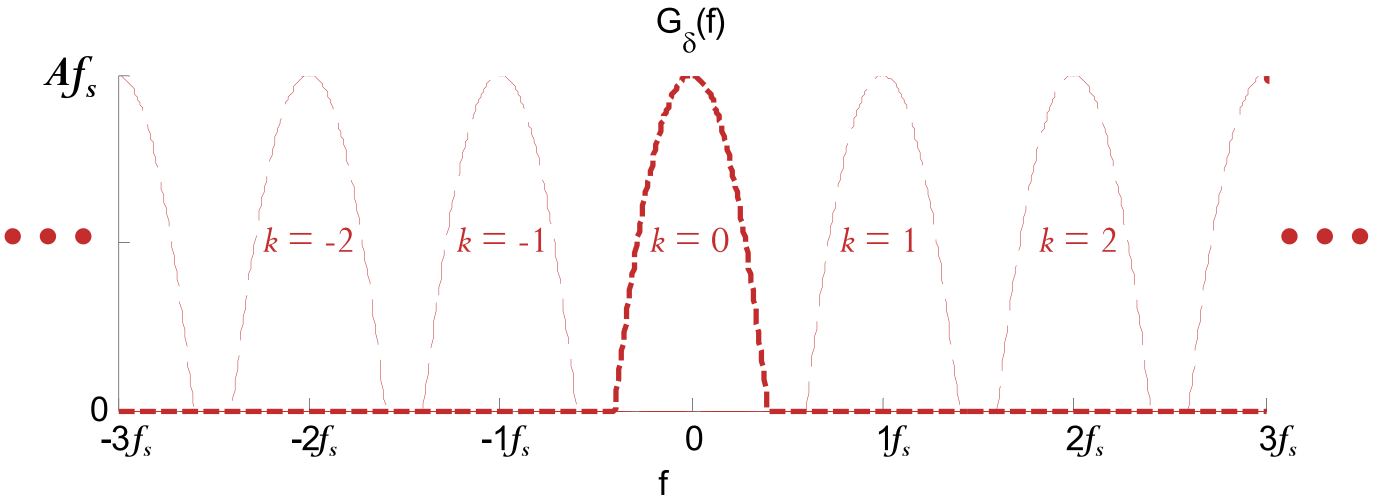
Ideal Sampling

$$G_\delta(f) = \sum_{k=-\infty}^{\infty} f_s G(f - kf_s)$$

The Fourier transform of the original signal

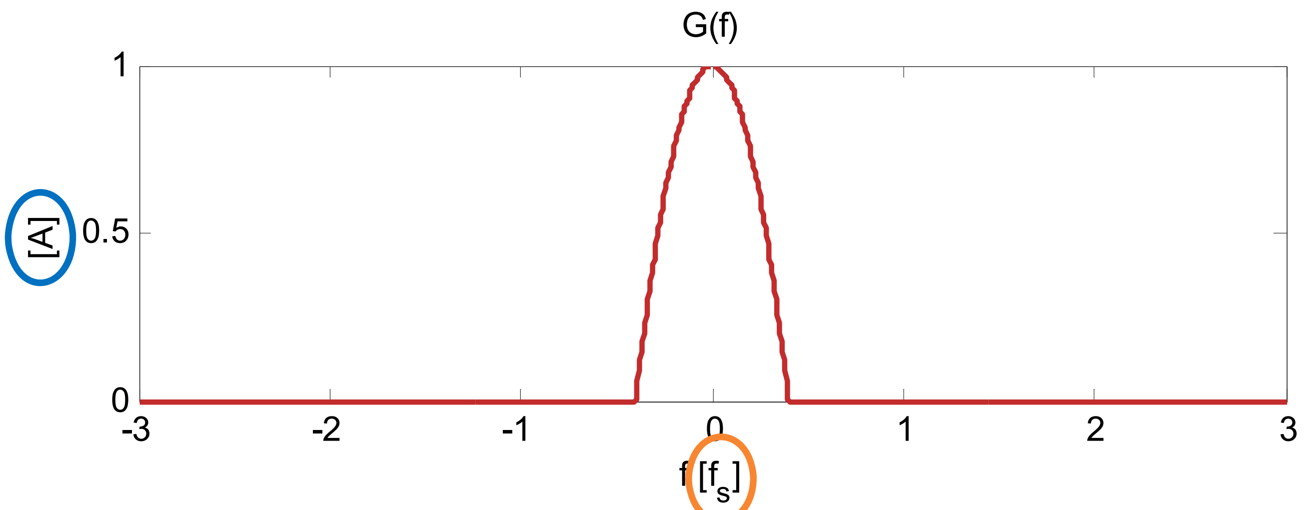


The Fourier transform of the (ideal) sampled signal

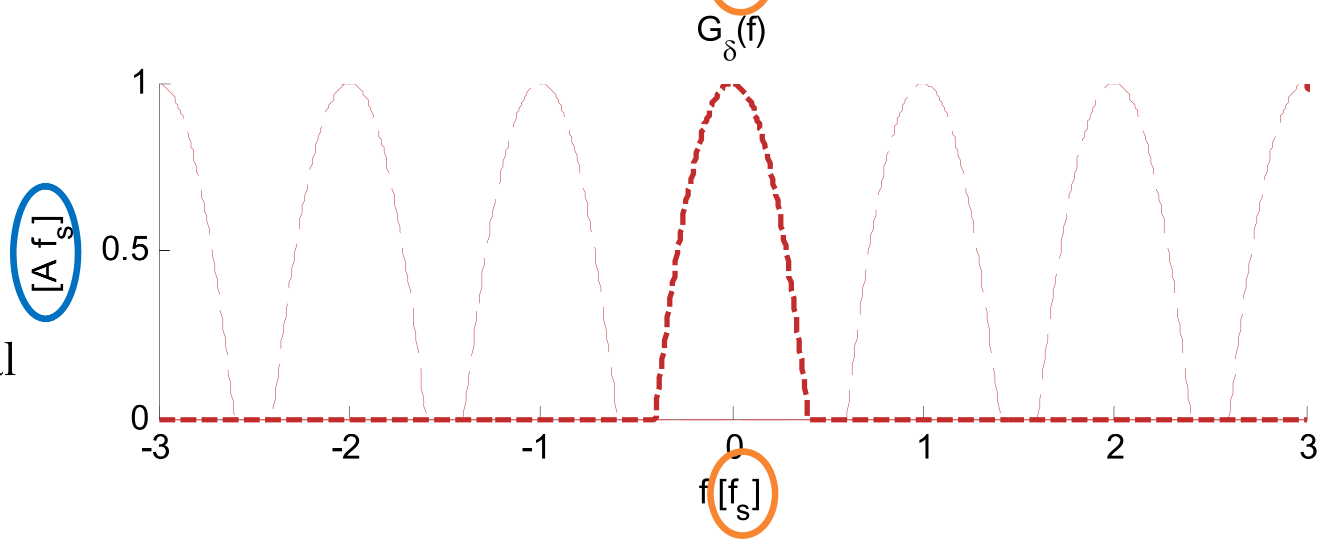


Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

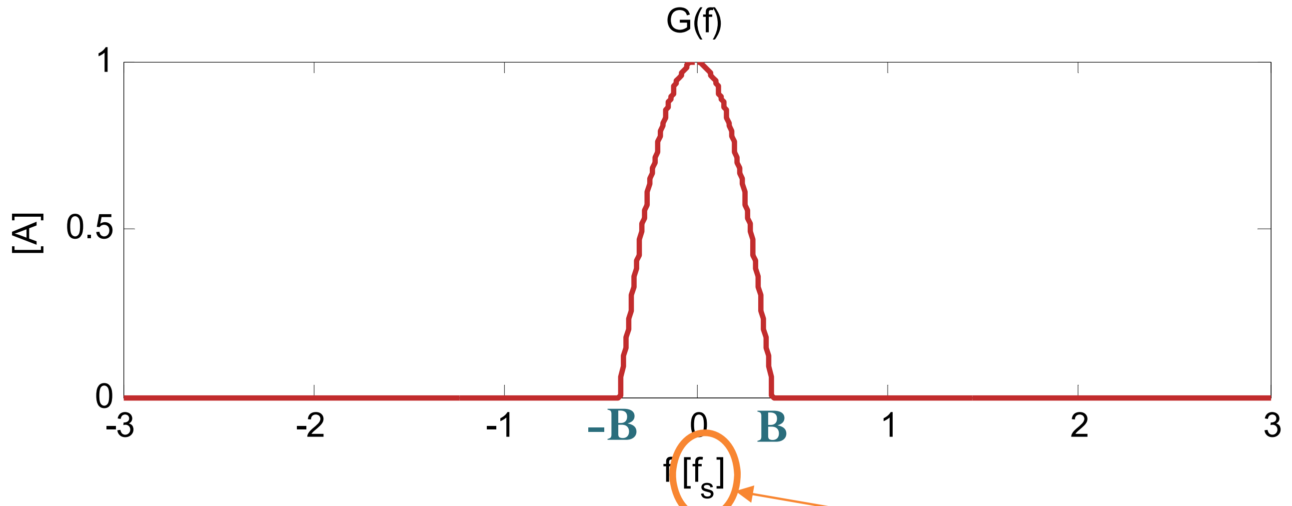


The Fourier transform of the (ideal) sampled signal

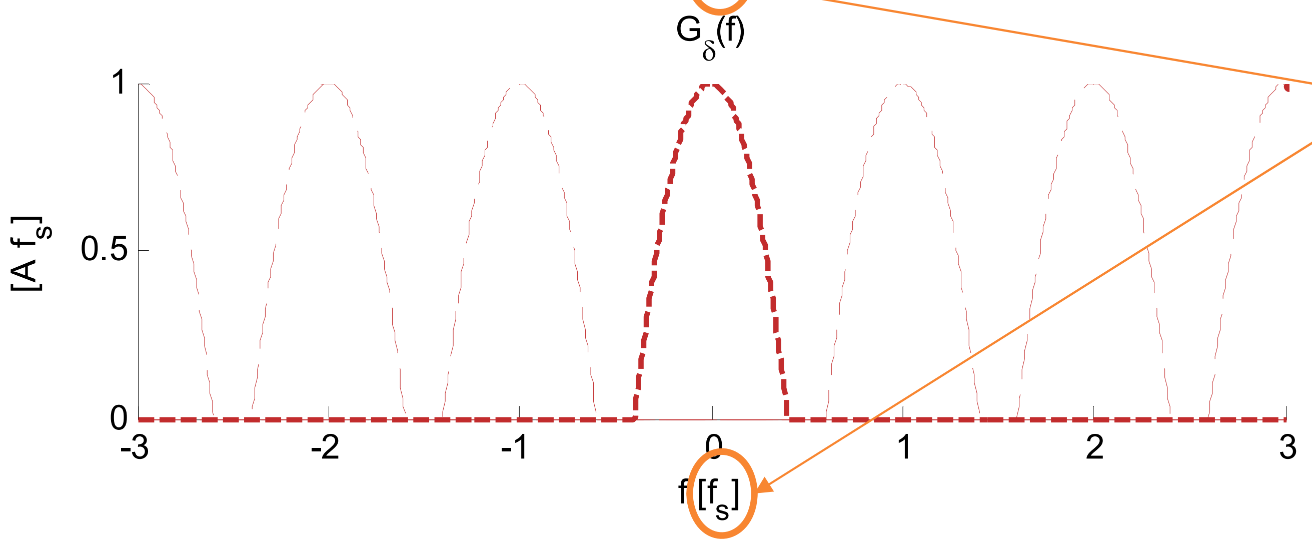


Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal



The Fourier transform of the (ideal) sampled signal

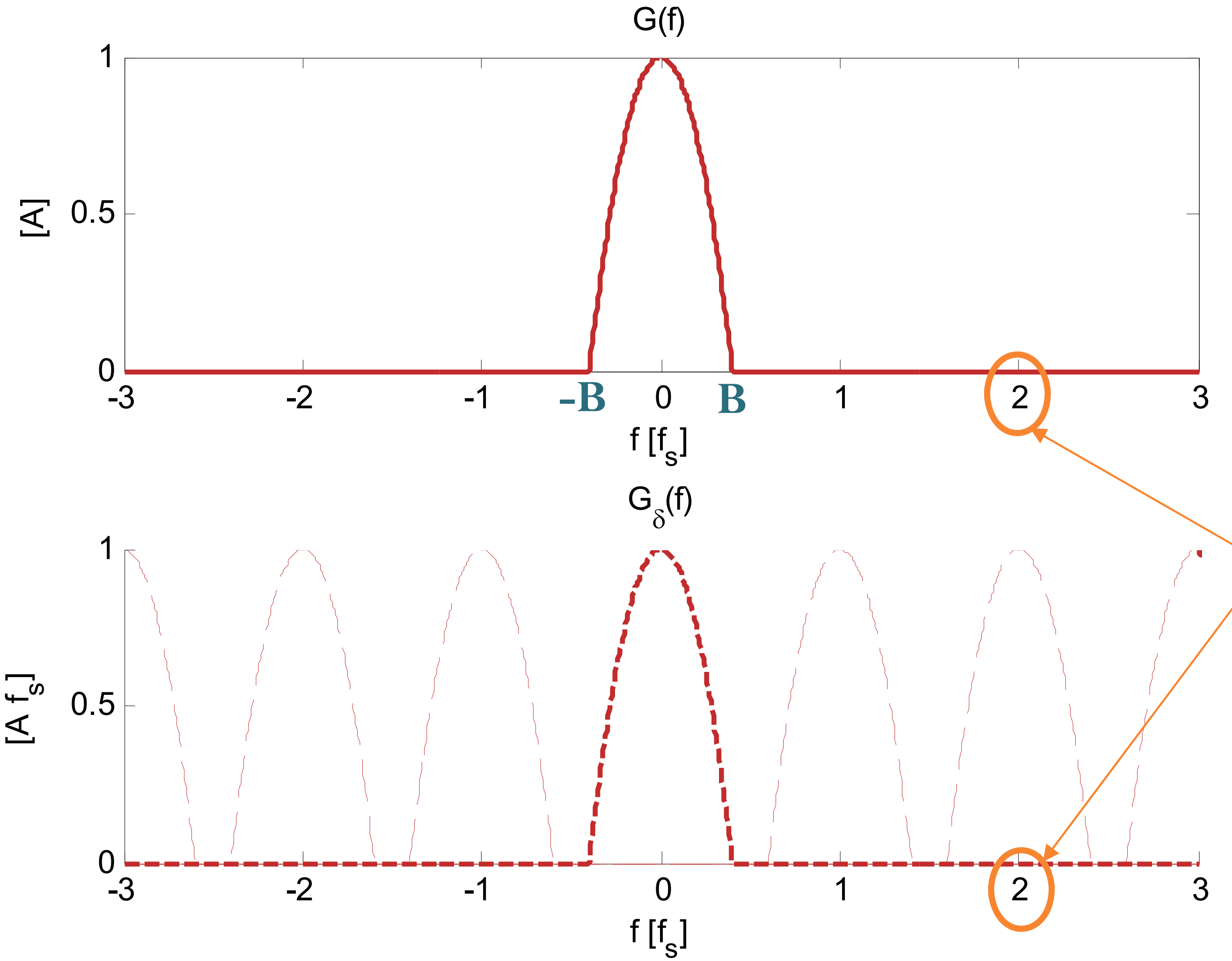


Note that the frequency unit here is f_s.

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal

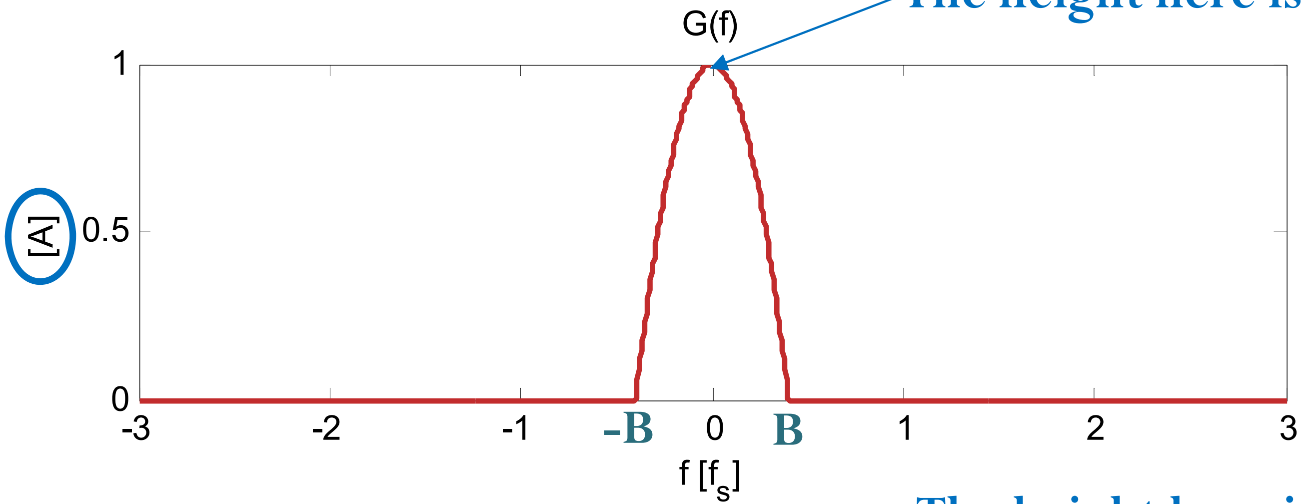


So, it's actually $2f_s$ here.

Ideal Sampling: MATLAB Exploration

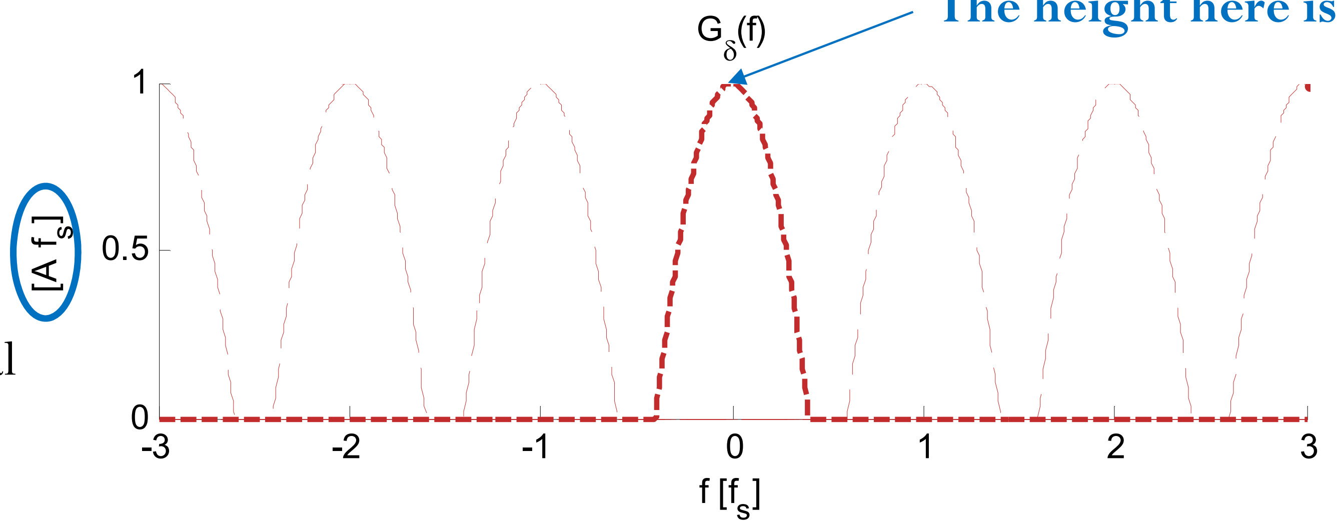
The height here is actually *A*.

The Fourier transform of the original signal



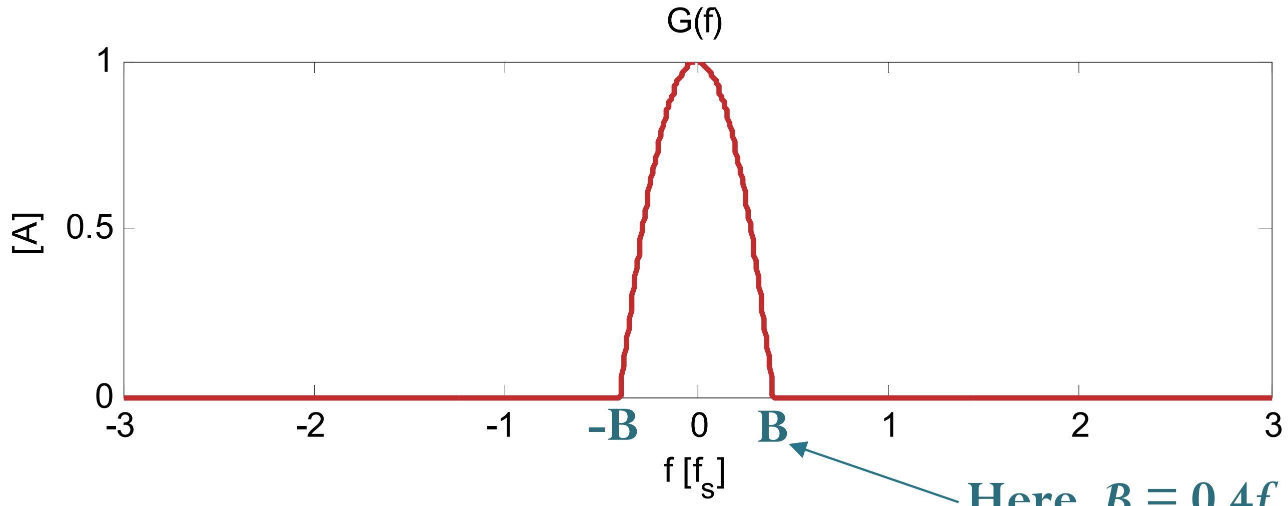
The height here is actually Af_s.

The Fourier transform of the (ideal) sampled signal



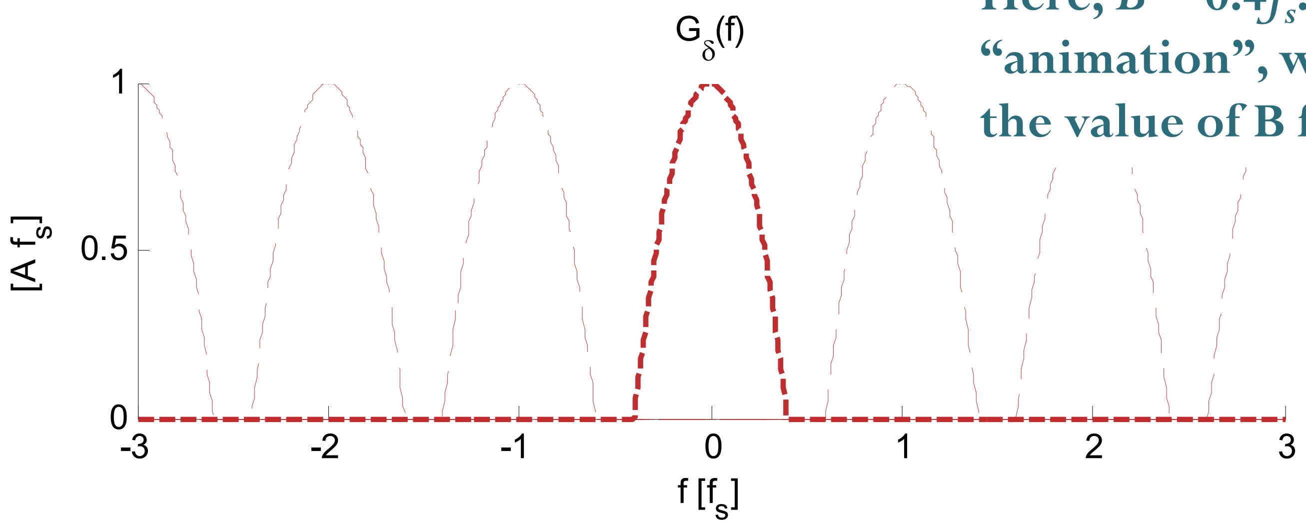
Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

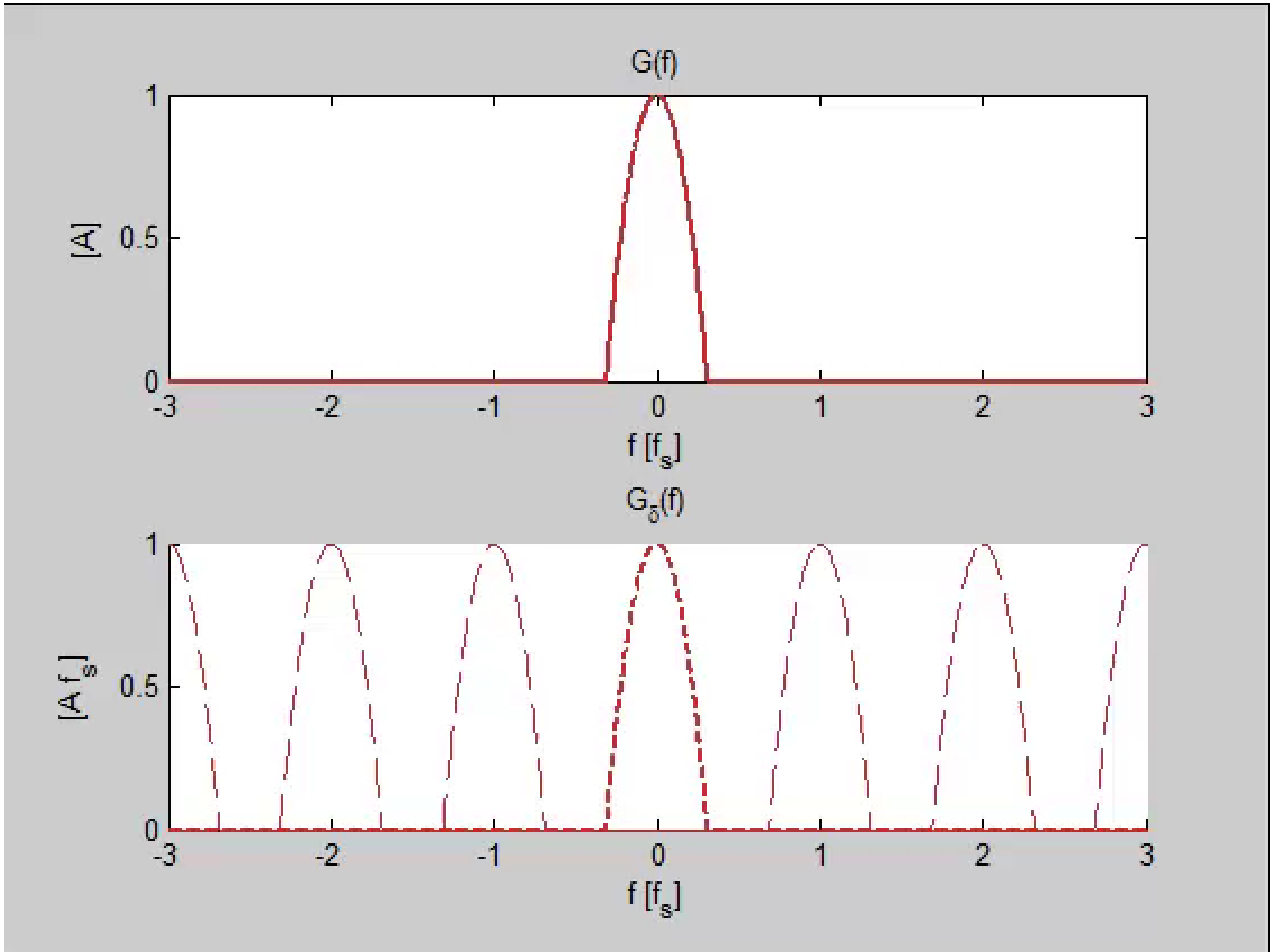


Here, $B = 0.4f_s$. In the "animation", we will increase the value of B from $0.2f_s$ to $3f_s$.

The Fourier transform of the (ideal) sampled signal



Ideal Sampling: MATLAB Exploration

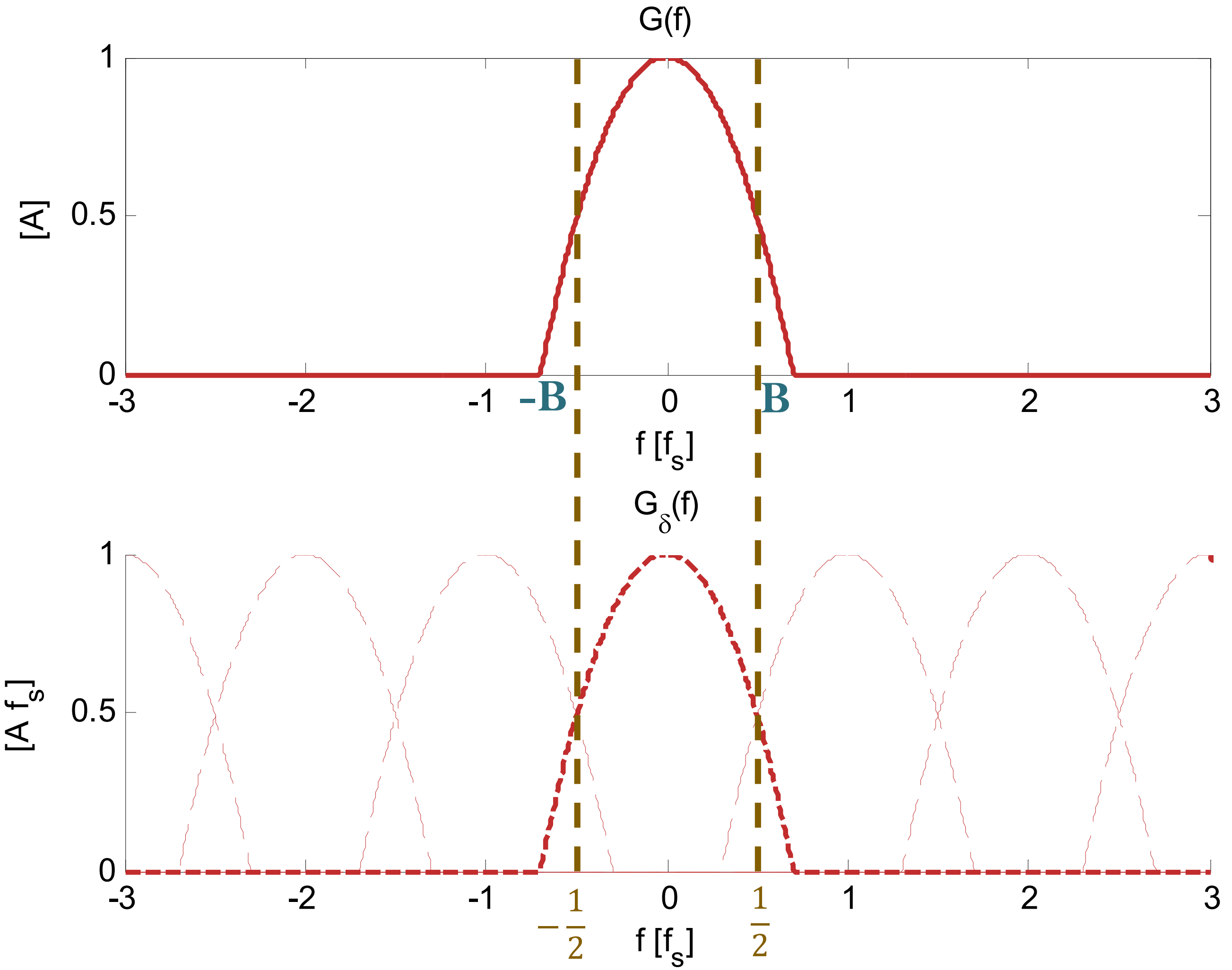


[Gdelta_demo1.m]

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal



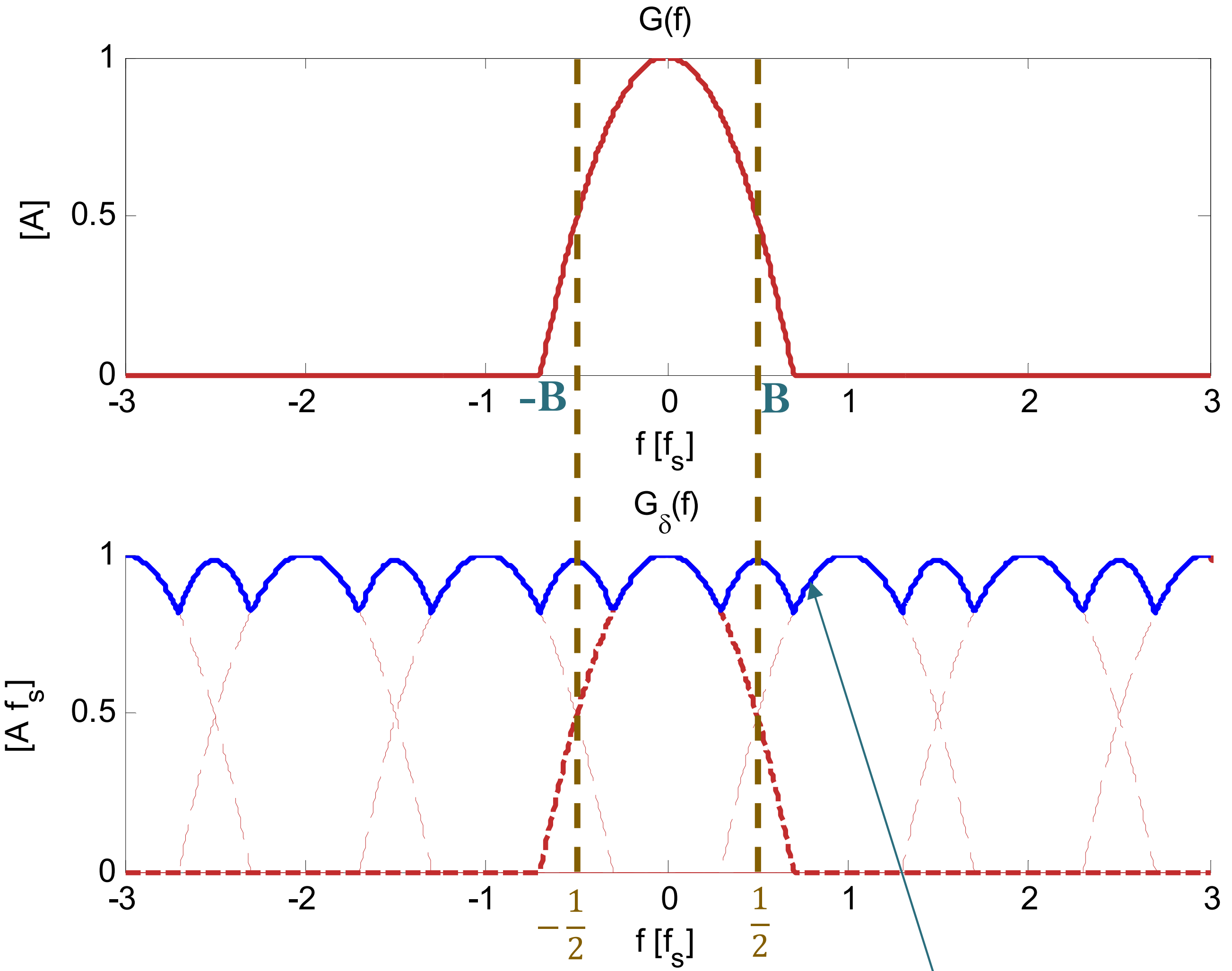
When B > fs/2, overlapping happens in the frequency domain.

This spectral overlapping of the signal is commonly referred to as "aliasing".

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

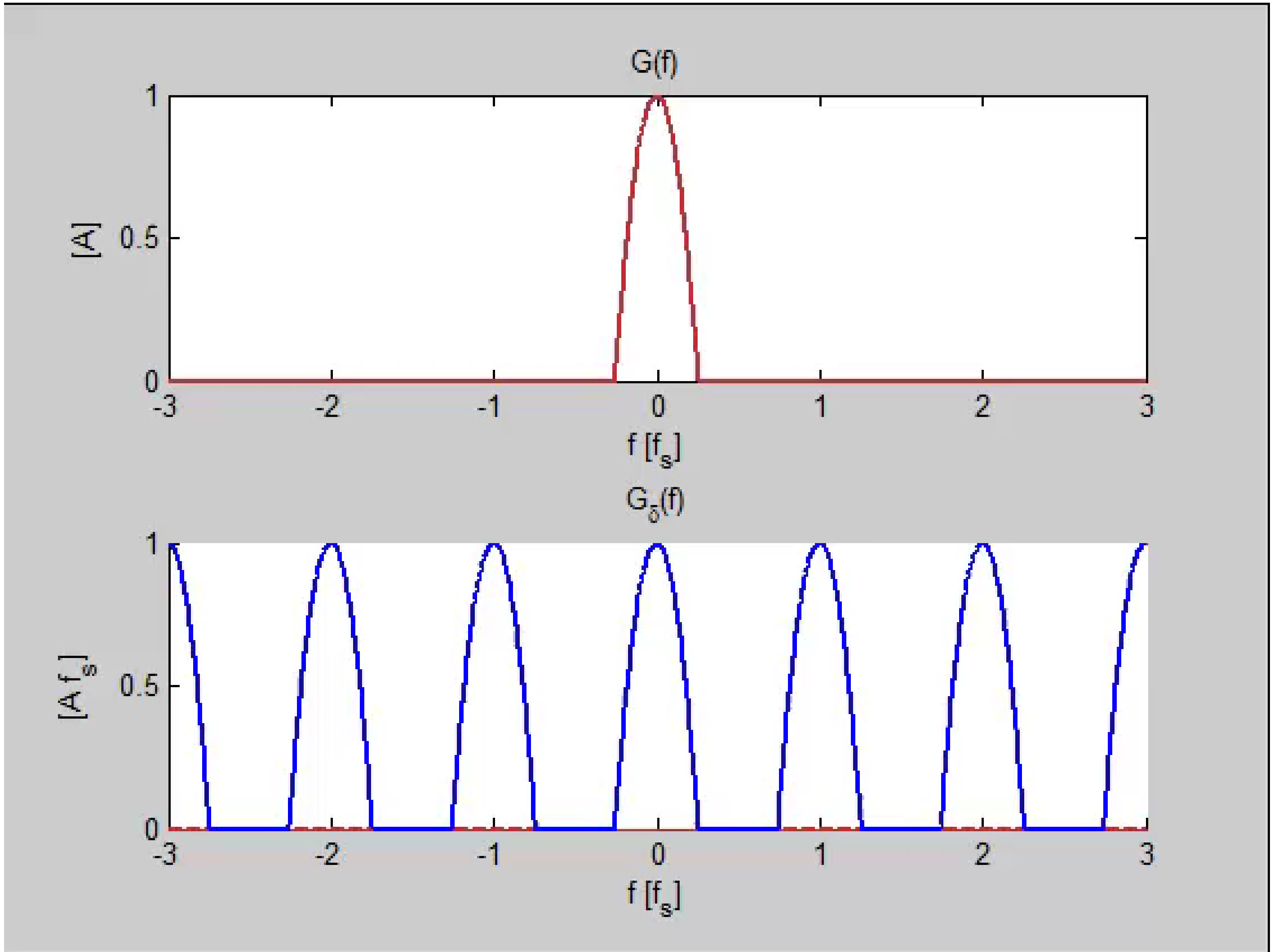
The Fourier transform of the (ideal) sampled signal



To find $G_\delta(f)$, don't forget to add the replicas

Very different from G(f).

Ideal Sampling: MATLAB Exploration

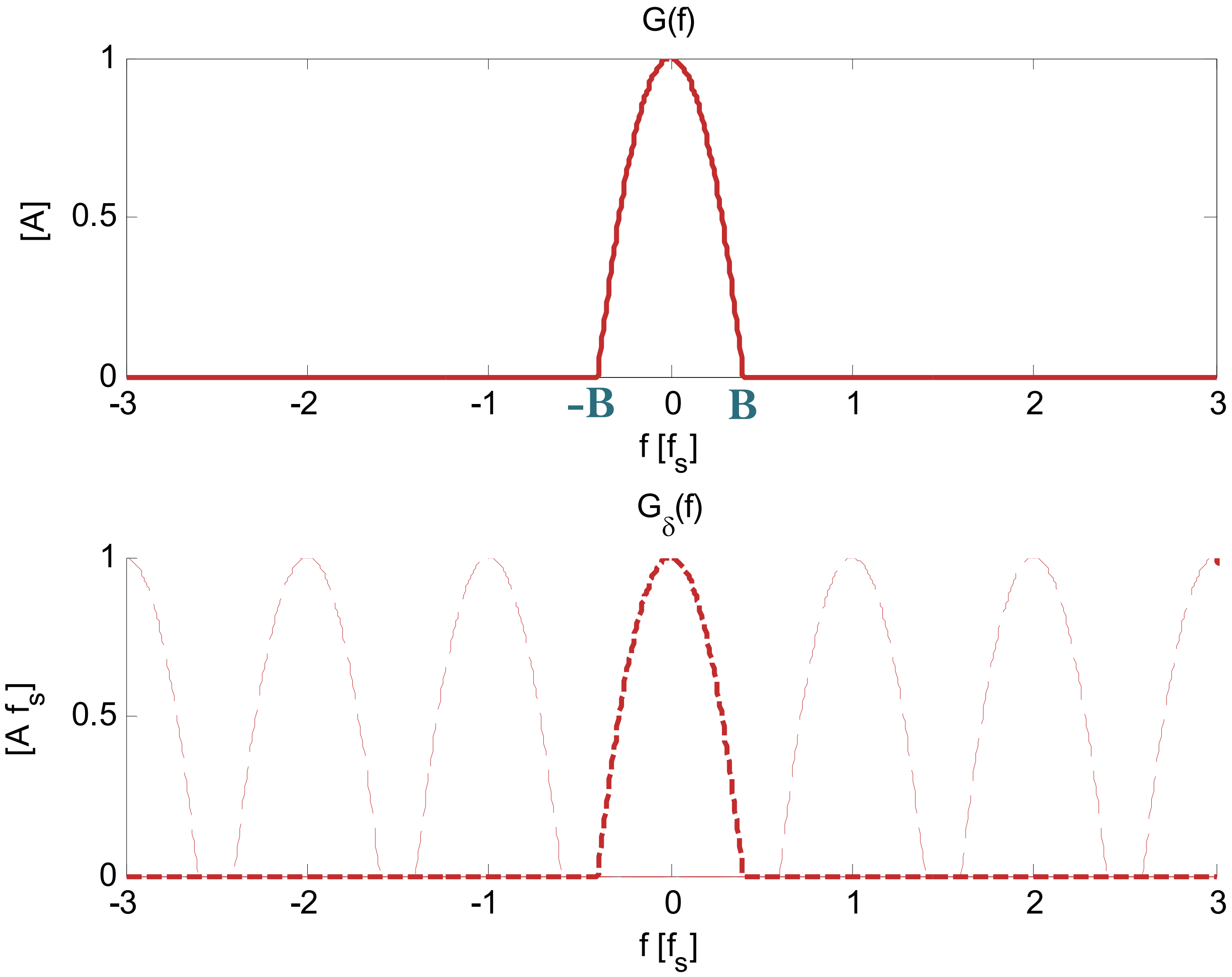


[Gdelta_demo2.m]

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal

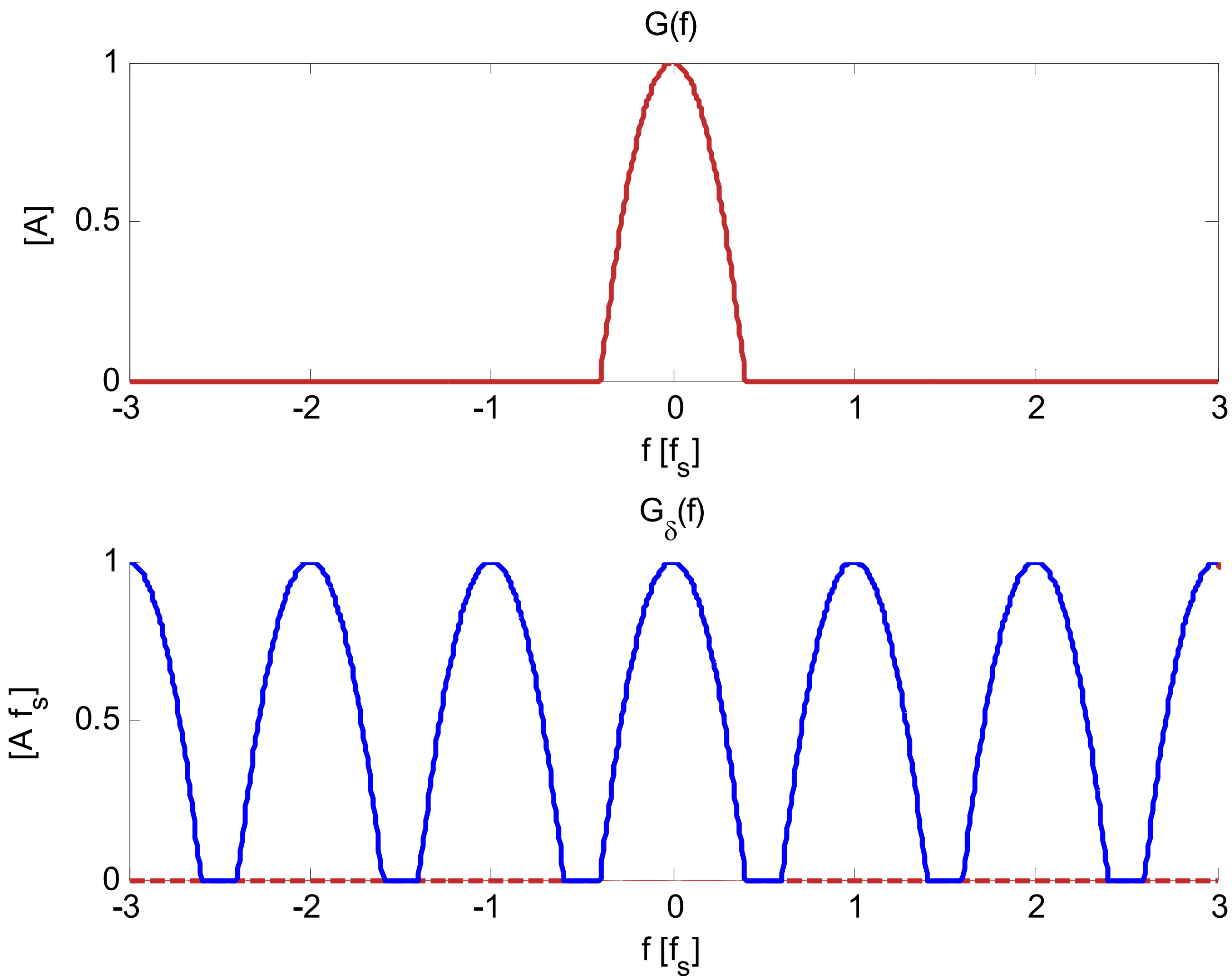


When $B < f_s/2$, the replicas do not overlap and hence we do not need to spend extra effort to find their sum.

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

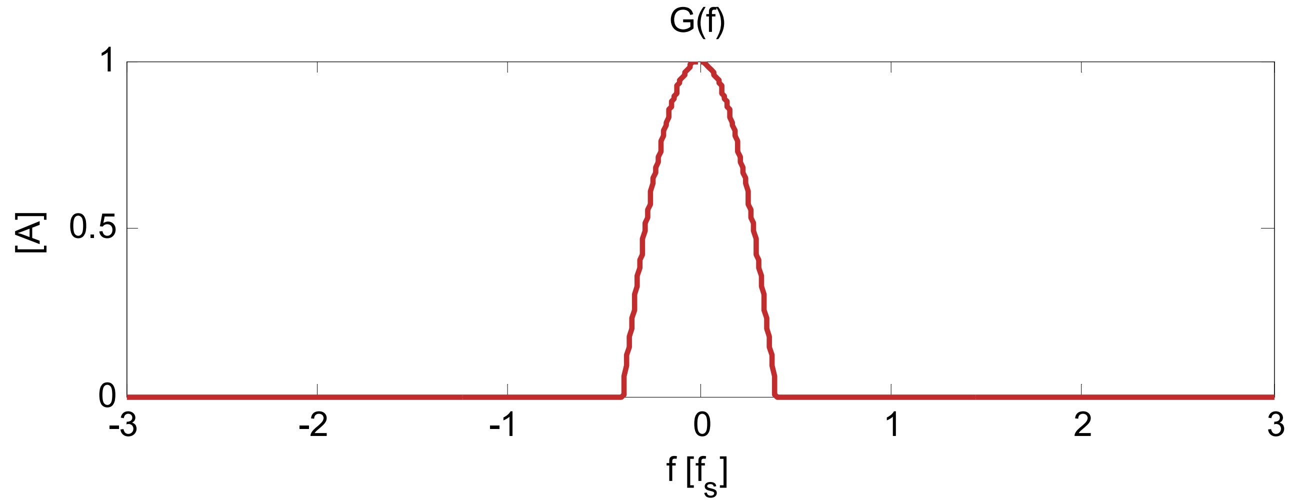
The Fourier transform of the (ideal) sampled signal



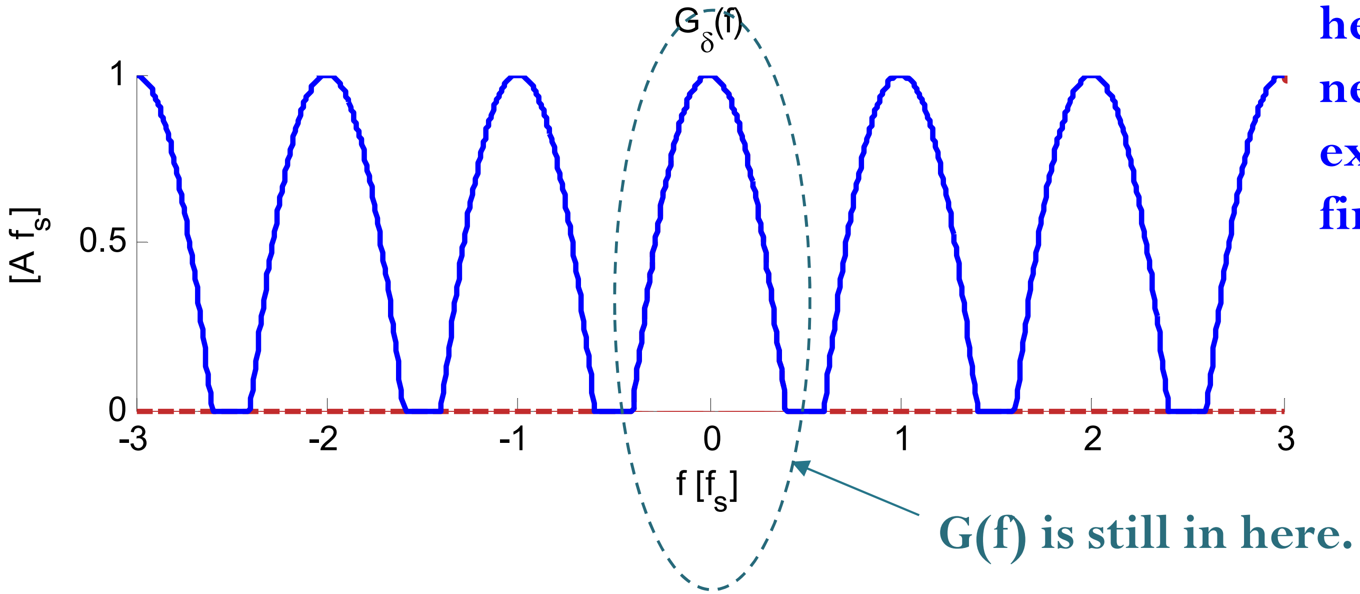
When $B < f_s/2$, the replicas do not overlap and hence we do not need to spend extra effort to find their sum.

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal



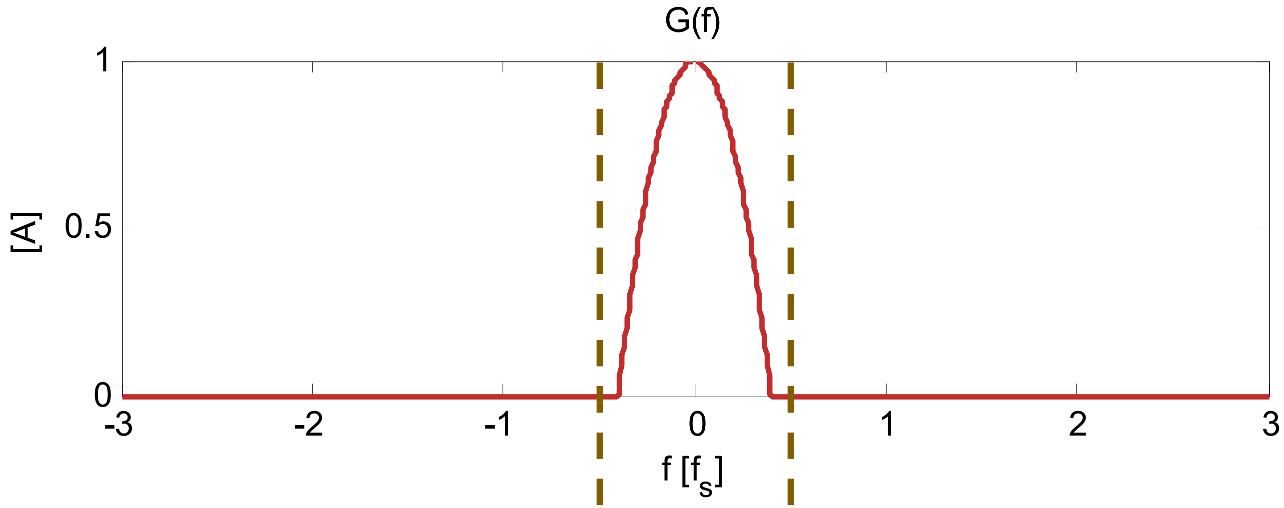
The Fourier transform of the (ideal) sampled signal



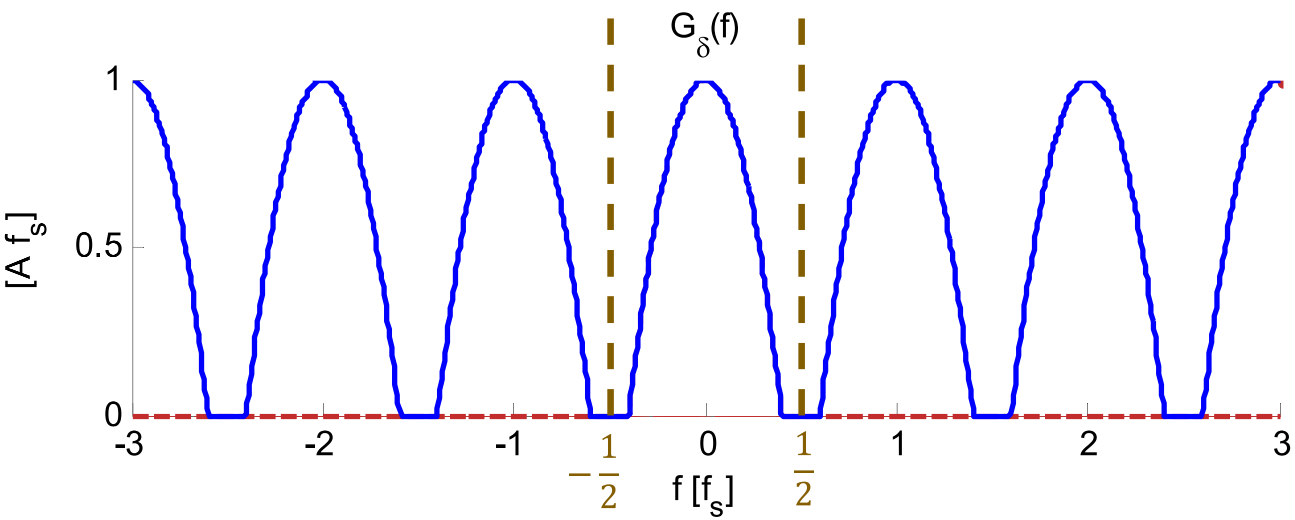
When B < fs/2, the replicas do not overlap and hence we do not need to spend extra effort to find their sum.

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal



The Fourier transform of the (ideal) sampled signal

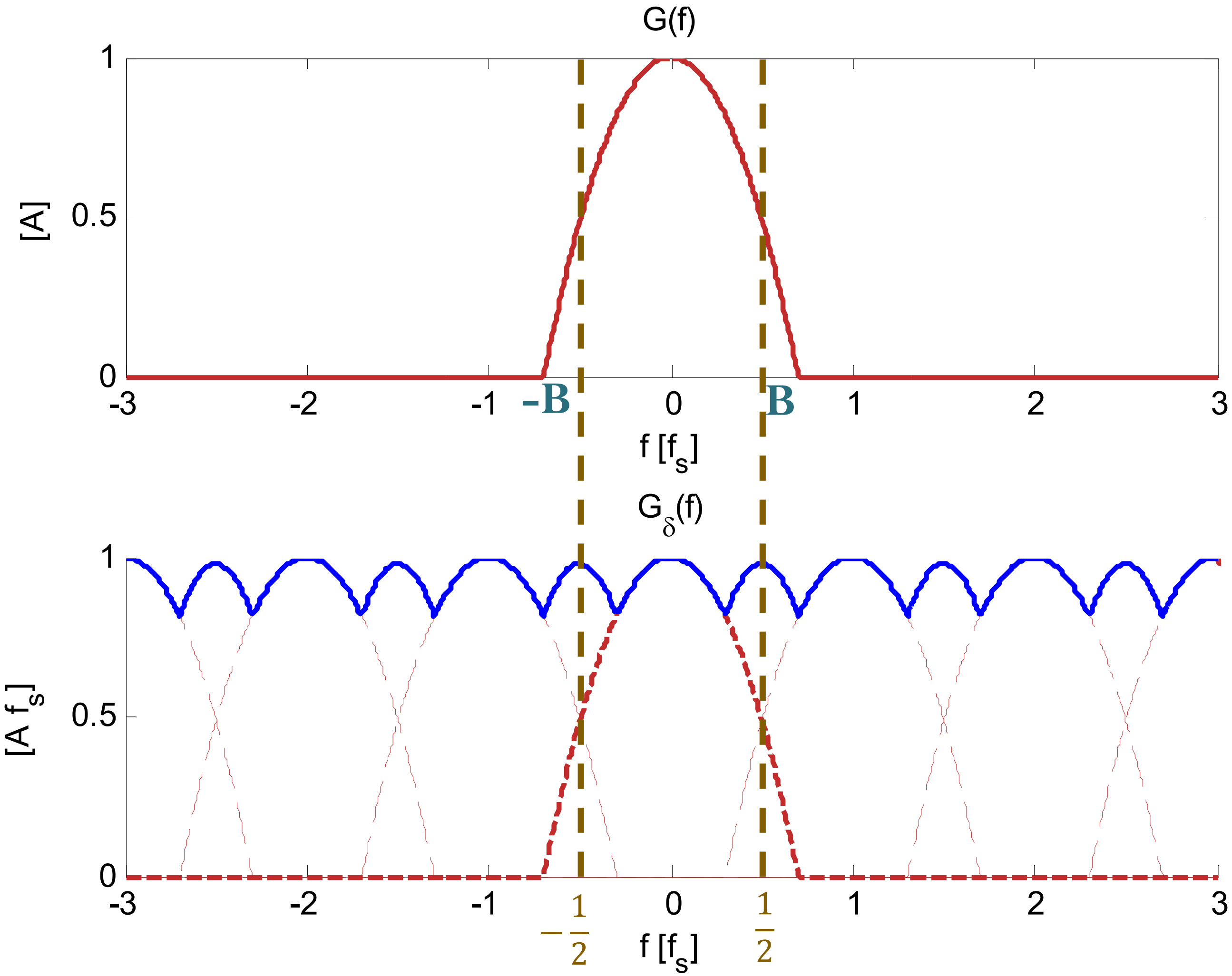


Note that $G_\delta(f)$ is "periodic" in the frequency domain with period f_s. Therefore, it is sufficient to look only at f between $\pm\frac{f_s}{2}$.

Ideal Sampling: MATLAB Exploration

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal



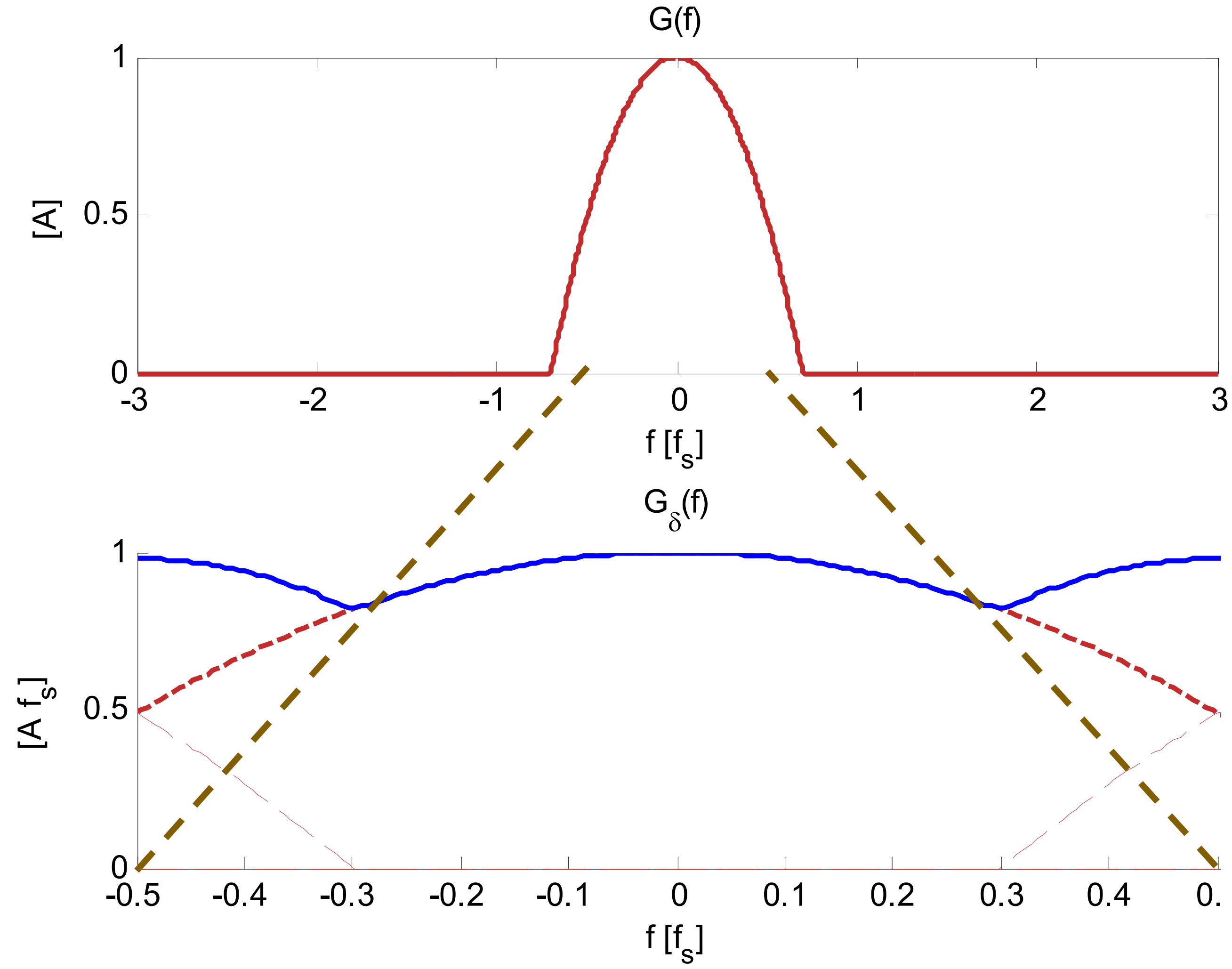
The periodicity holds regardless of whether the replicas overlap in the frequency domain.

Note that $G_\delta(f)$ is "periodic" in the frequency domain with period f_s. Therefore, it is sufficient to look only at f between $\pm\frac{f_s}{2}$.

Ideal Sampling: plotspect's view

- The function **`plotspect`** relies on the sampled version of the signal.
- Any corruption of information (aliasing) from the sampling process will also be "visible" in the output of **`plotspect`**.
- **`plotspect`** also looks only at f between $\pm \frac{f_s}{2}$.
  - With some vertical scaling.

Ideal Sampling: plotspect's view

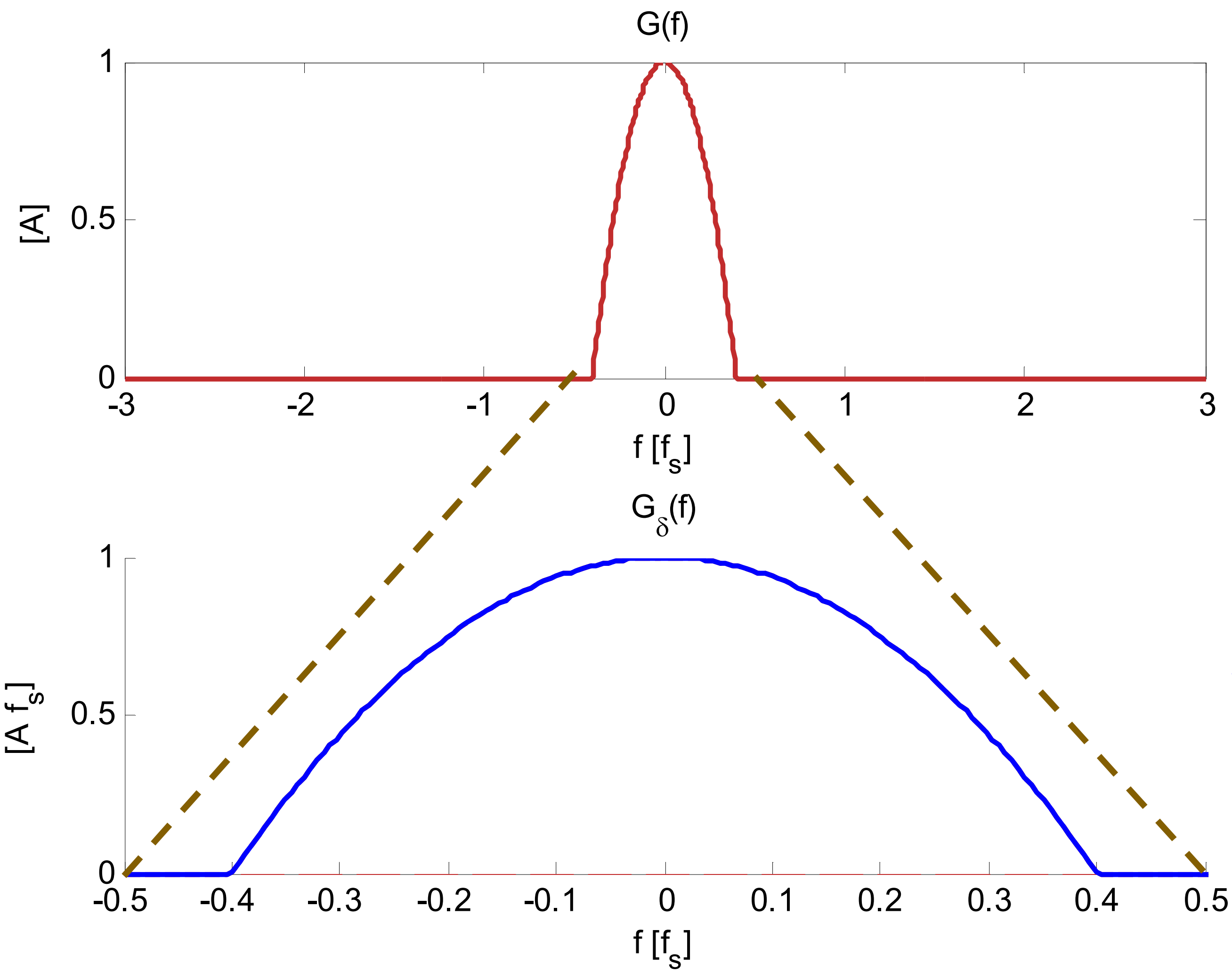


When $B > f_s/2$, **`plotspect`'s** result will be quite different from the expected theoretical/analytical Fourier transform $G(f)$.

Ideal Sampling: plotspect's view

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal



For `plotspect` to give an accurate view, we need B < fs/2.

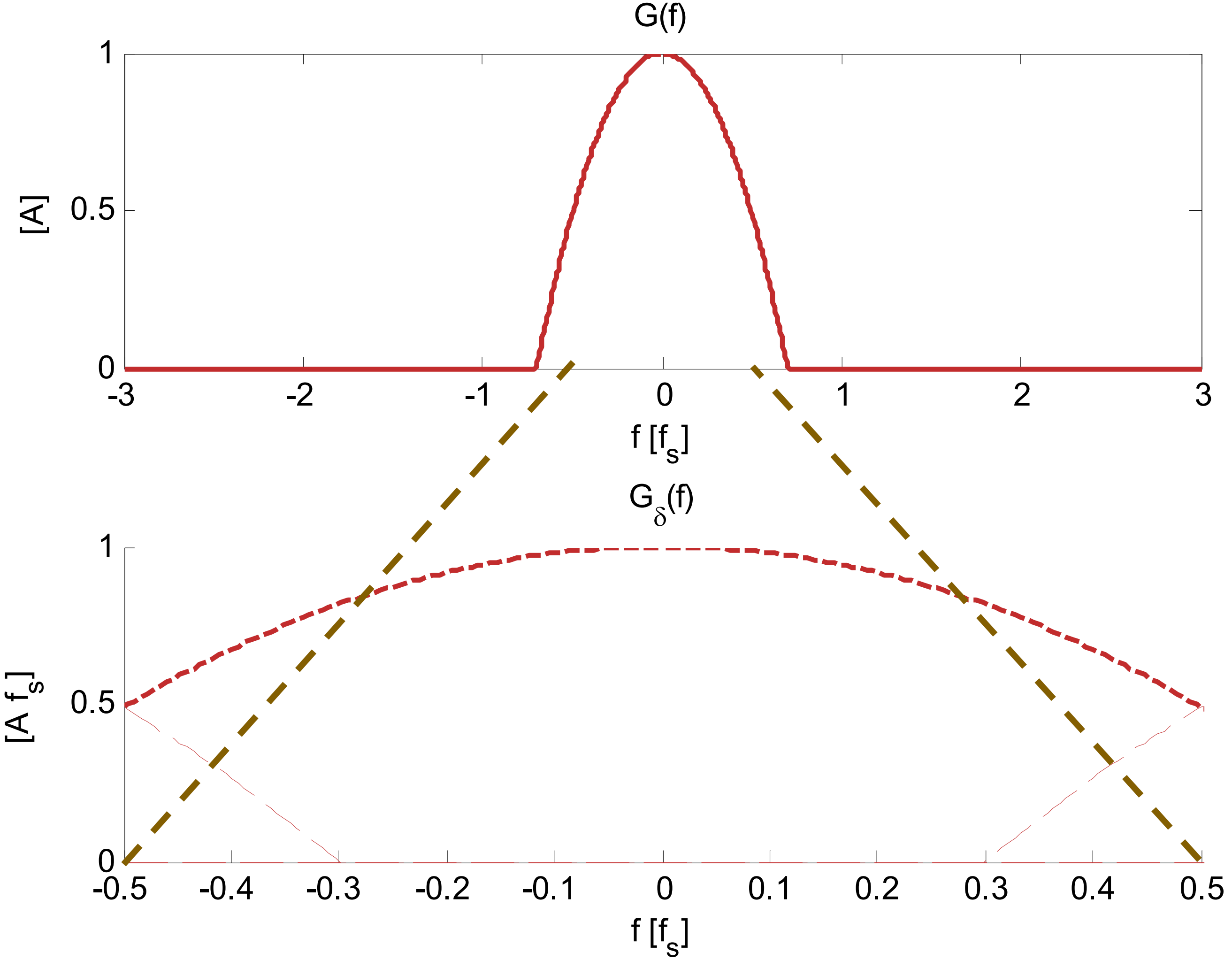
Ideal Sampling: Folding

- When the signal $g(t)$ is **real-valued**, recall that its Fourier transform has conjugate symmetry.
- It is sufficient to look at the positive frequency if we **care only about the magnitude**.
- Therefore, we can limit our view to $[0, f_s/2]$.

Ideal Sampling: from $-f_s/2$ to $f_s/2$

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal

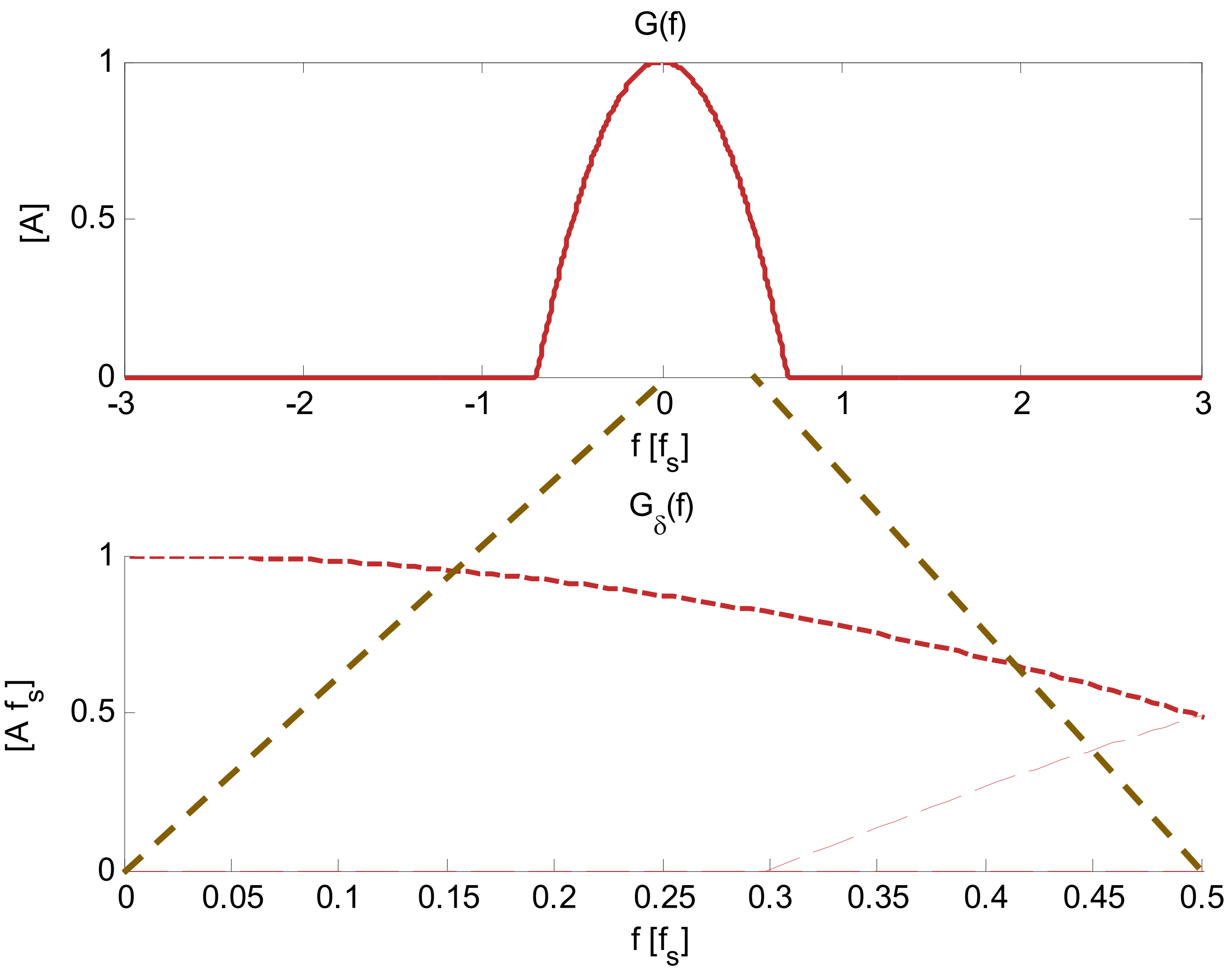


Note the symmetry.

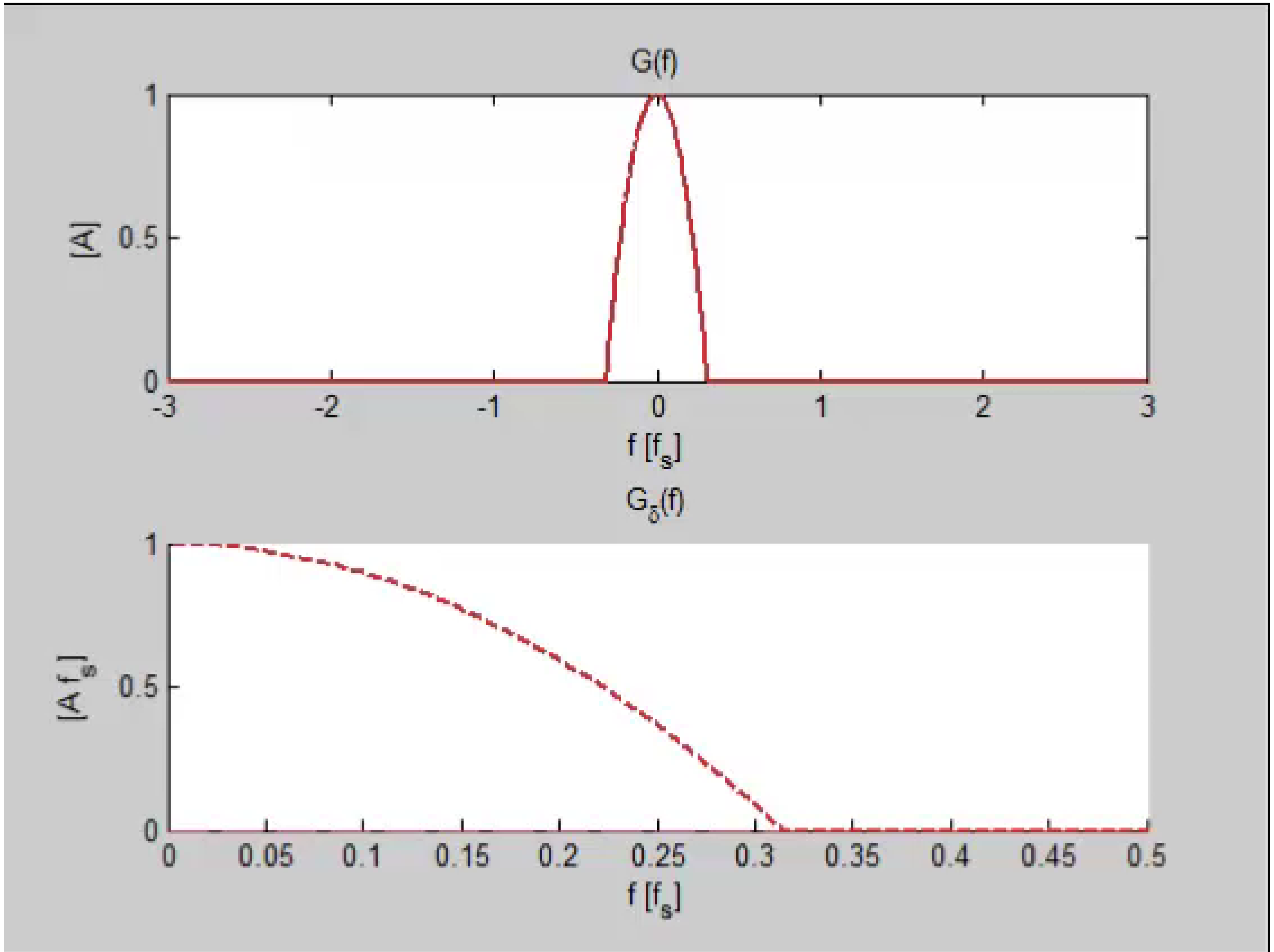
Ideal Sampling: Folding

The Fourier transform of the original signal

The Fourier transform of the (ideal) sampled signal

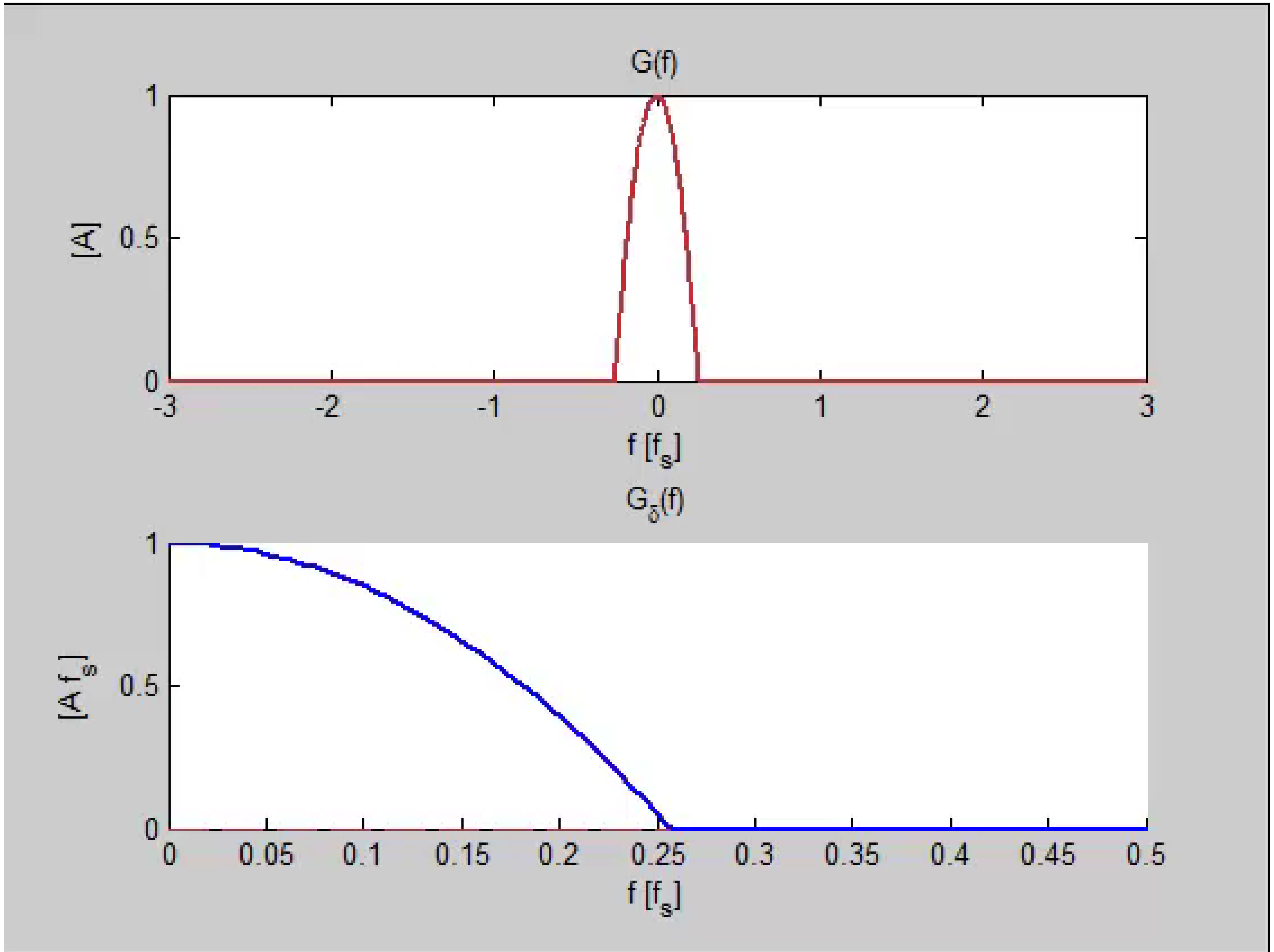


Ideal Sampling: Folding



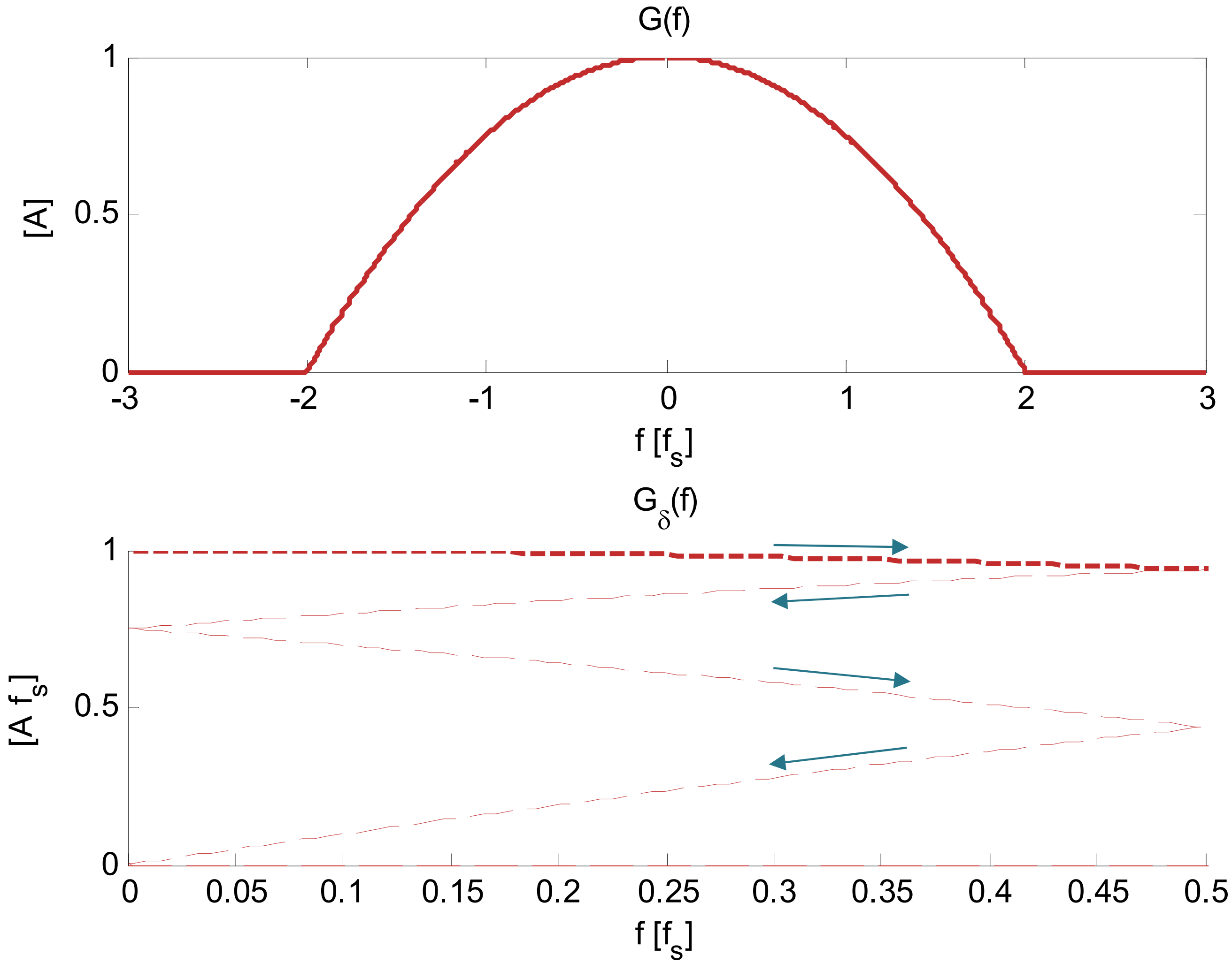
[Gdelta_demo3.m]

Ideal Sampling: Folding



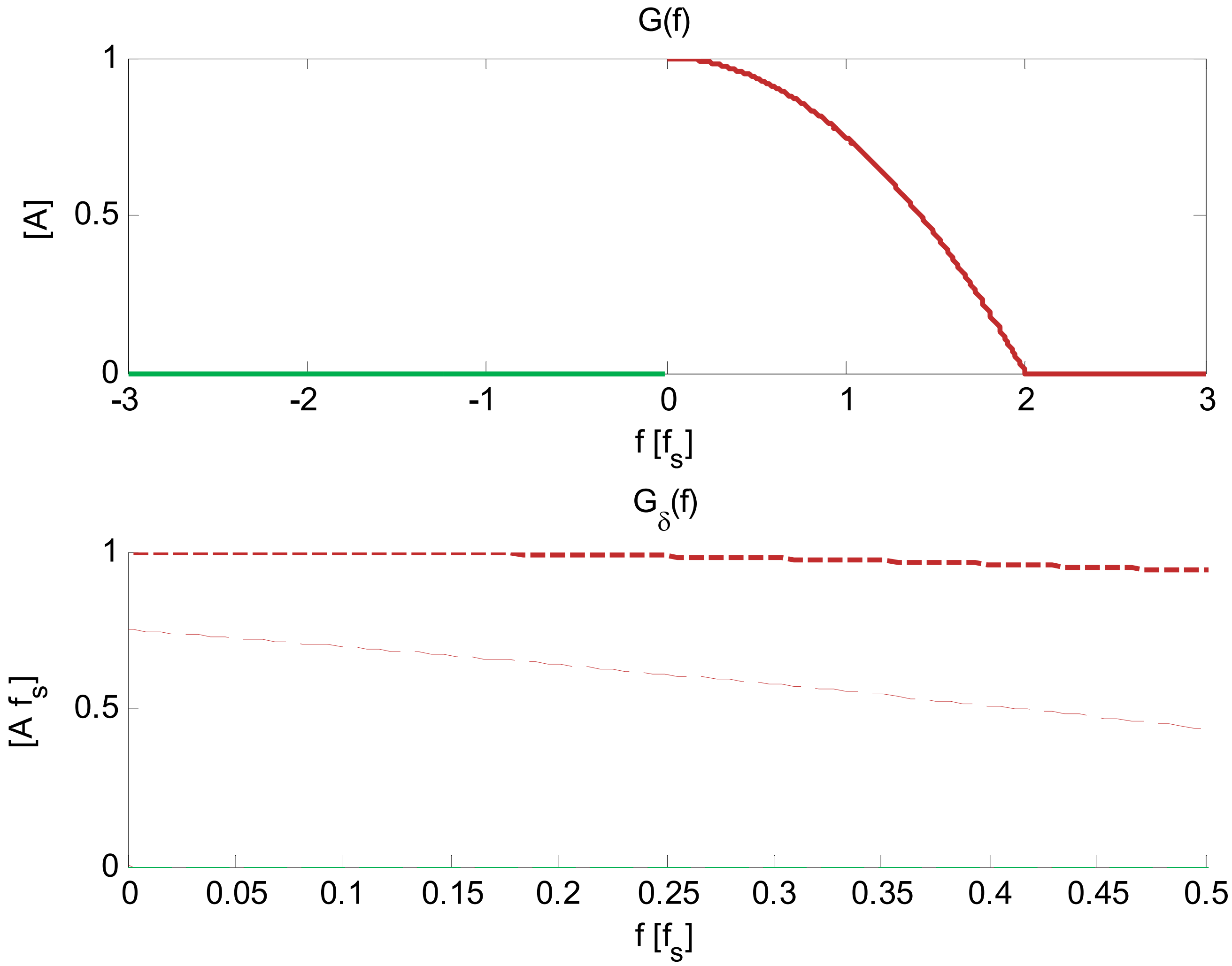
[Gdelta_demo4.m]

Ideal Sampling: Folding



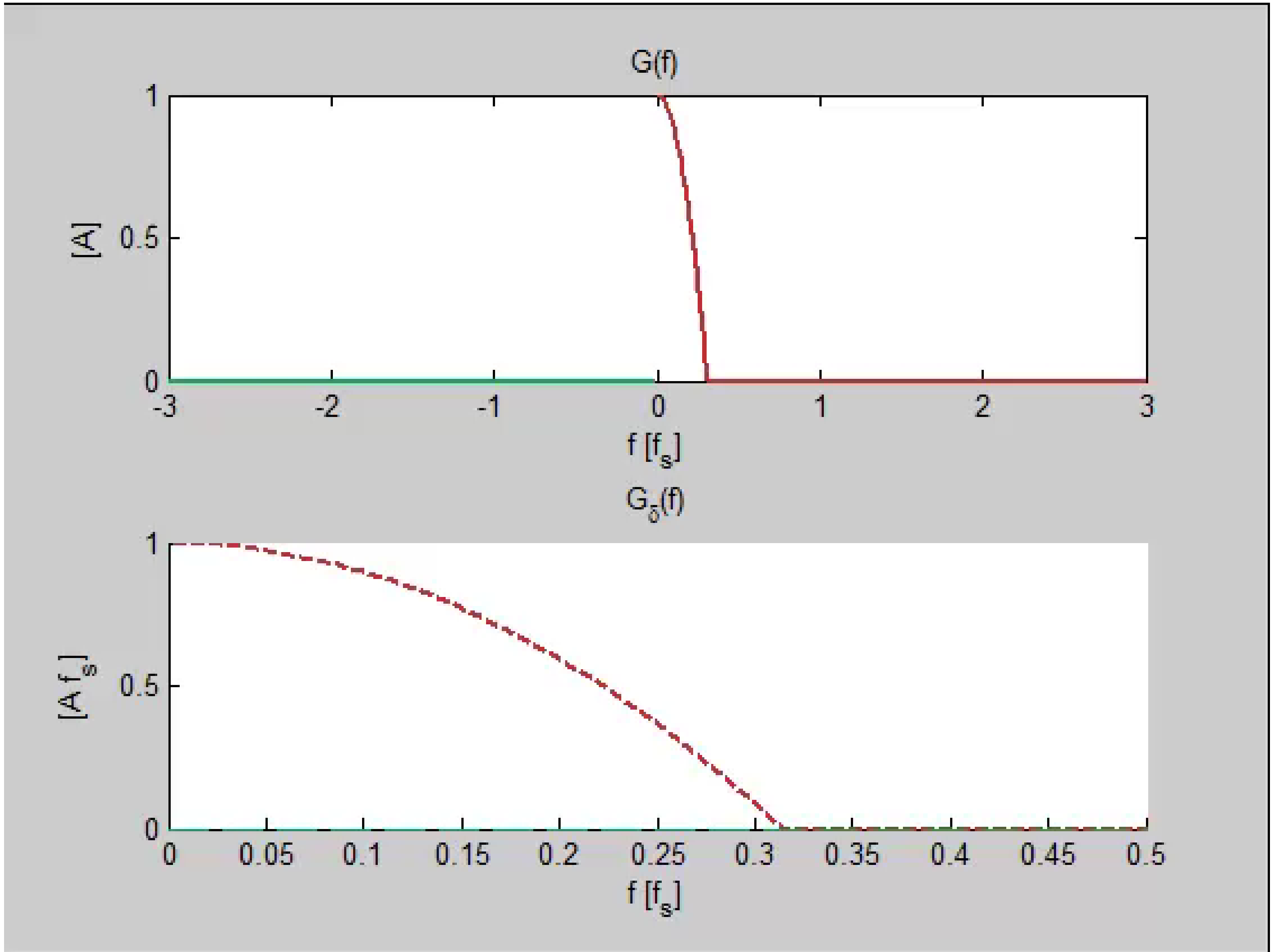
The animation did make it look like $G(f)$ is being folded repeatedly at 0 and $f_s/2$. This is correct as long as 1) you are assuming real-valued $g(t)$ and 2) you only want to look at the magnitude of $G(f)$.

Ideal Sampling: Folding



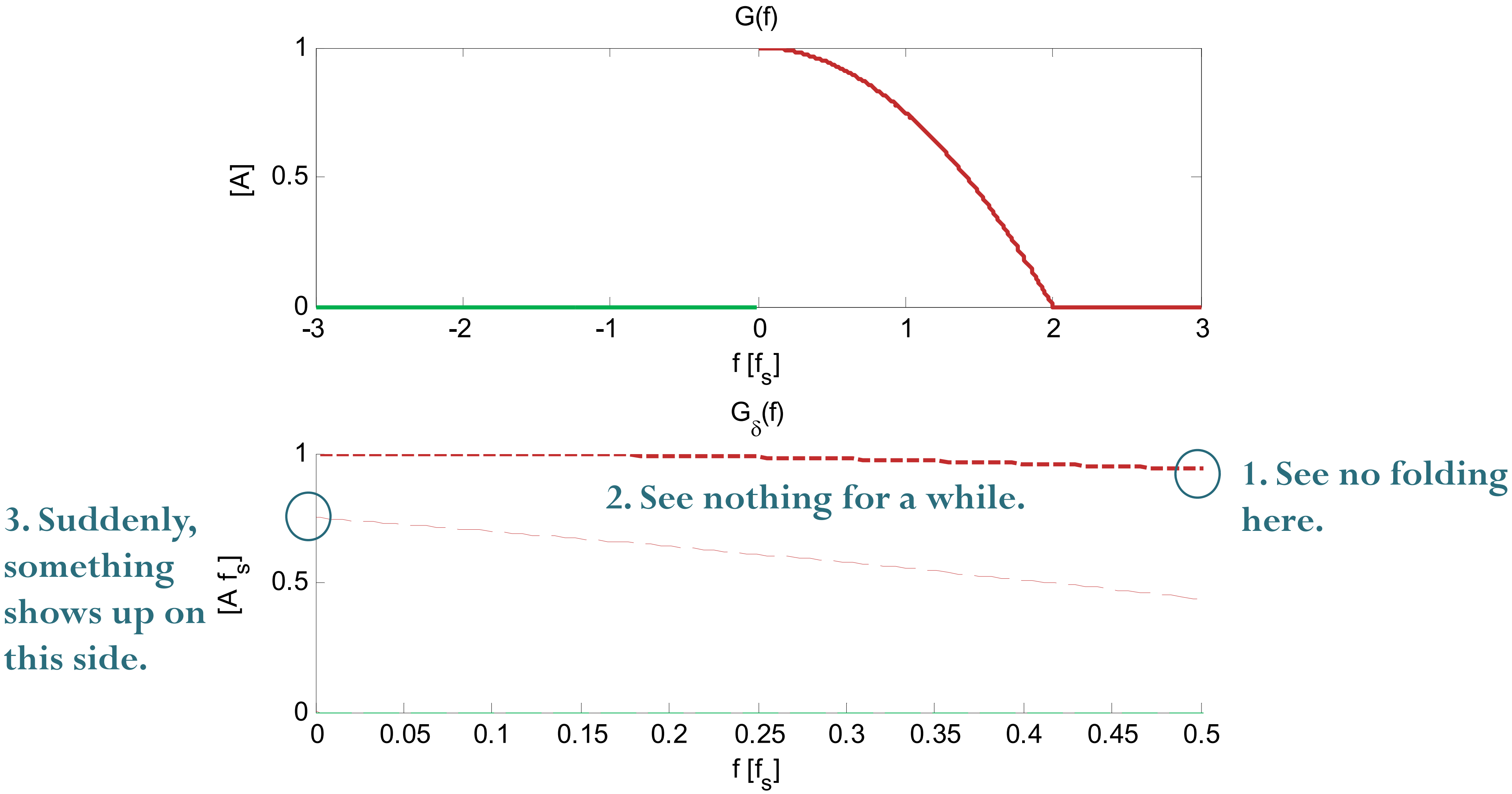
When $G(f)$ resides only in the positive frequency, we start seeing the **flaw** of the "folding technique".

Ideal Sampling: Tunneling

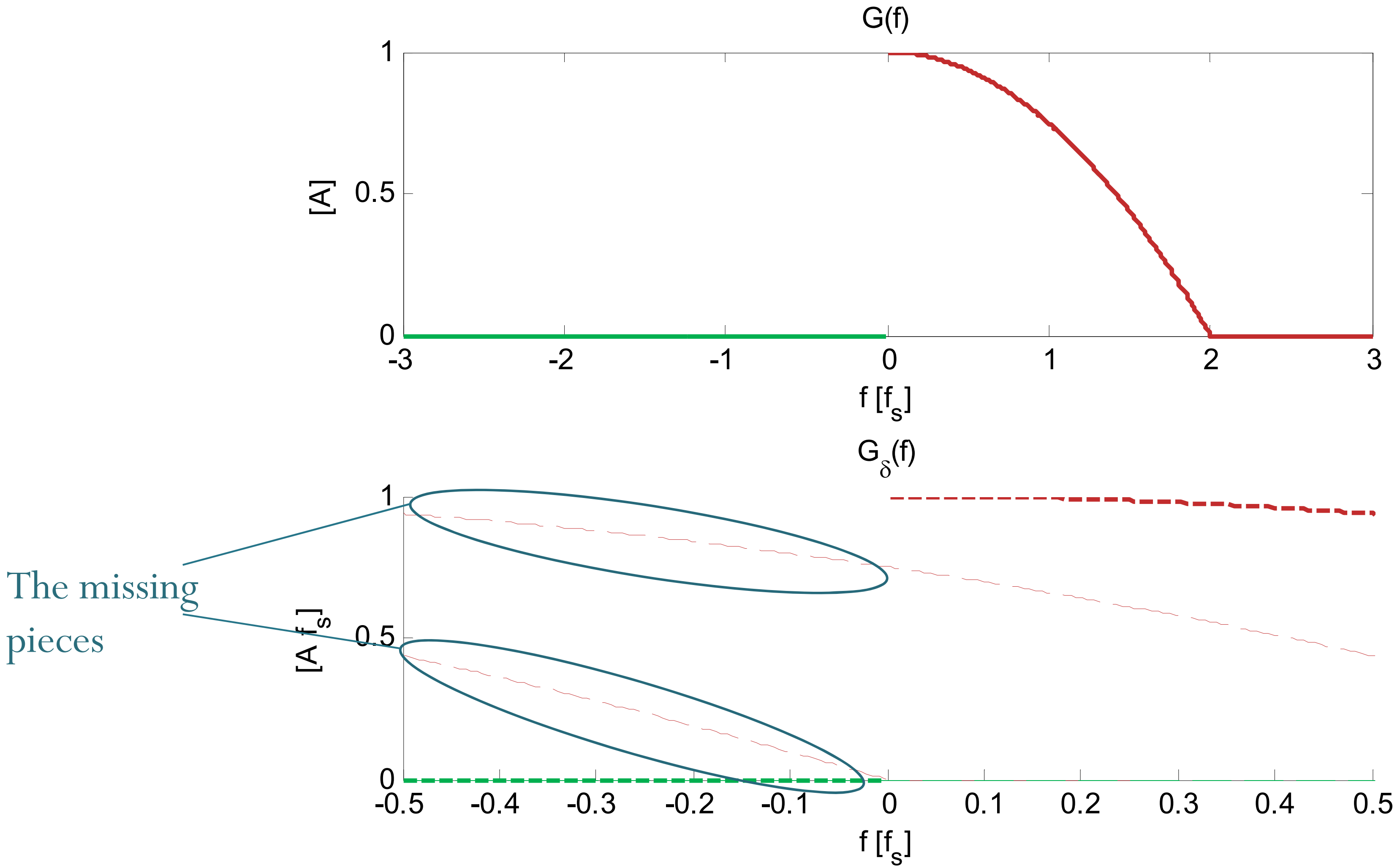


[Gdelta_demo6.m]

Ideal Sampling: Folding

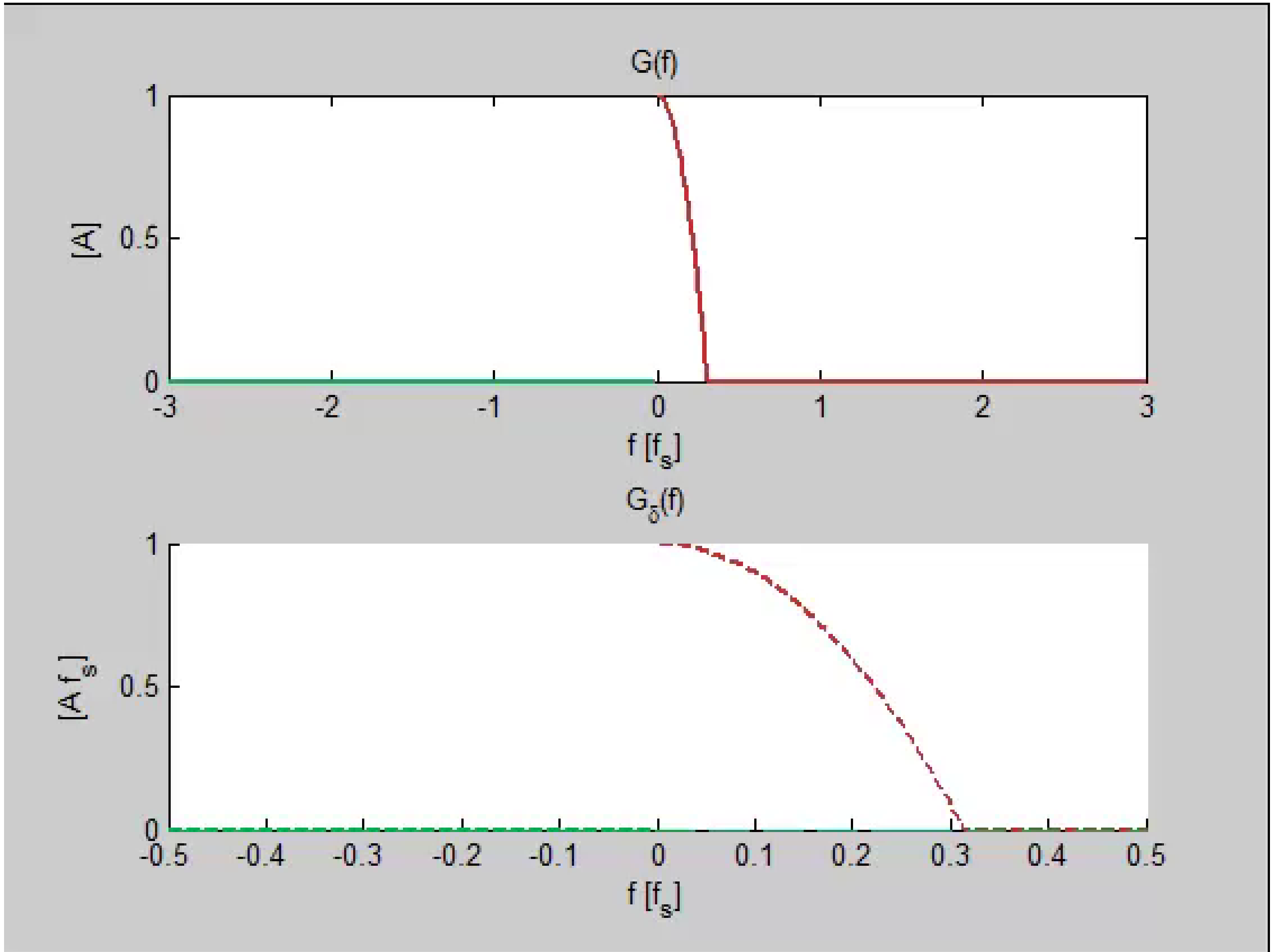


Ideal Sampling: Folding



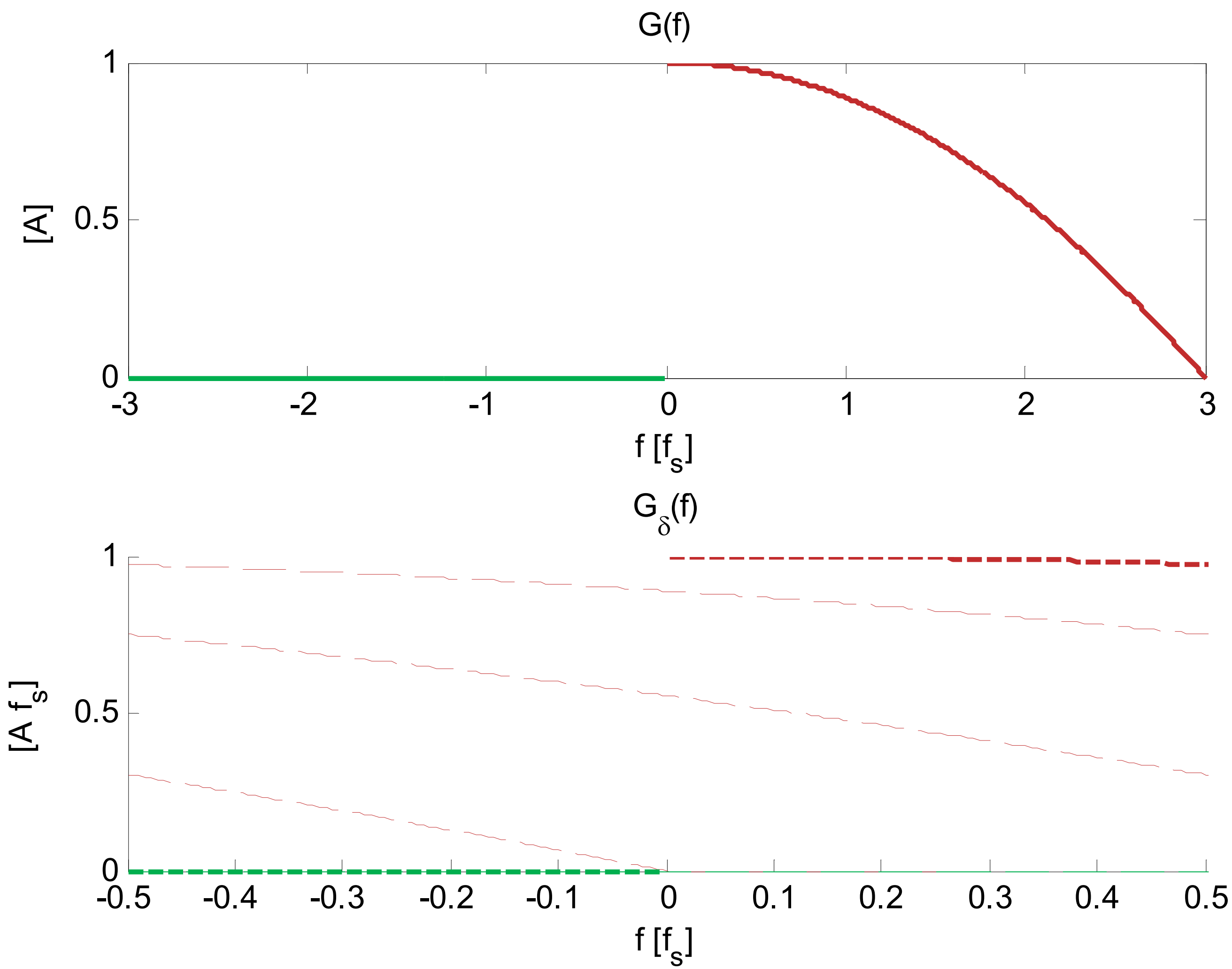
To see the complete picture, we should look at $G(f)$ between $\pm\frac{f_s}{2}$.

Ideal Sampling: Tunneling

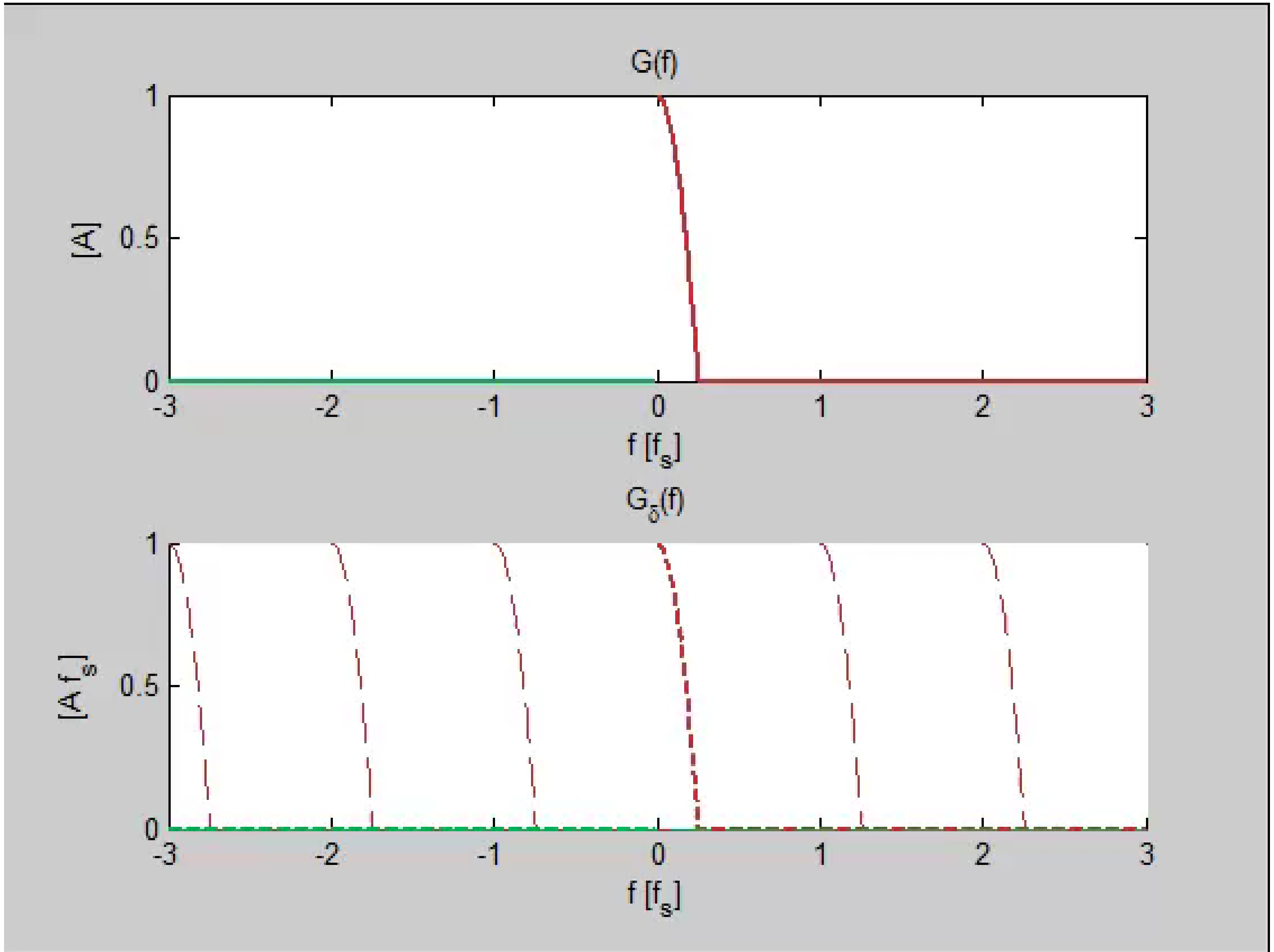


[Gdelta_demo5.m]

Ideal Sampling: Tunneling

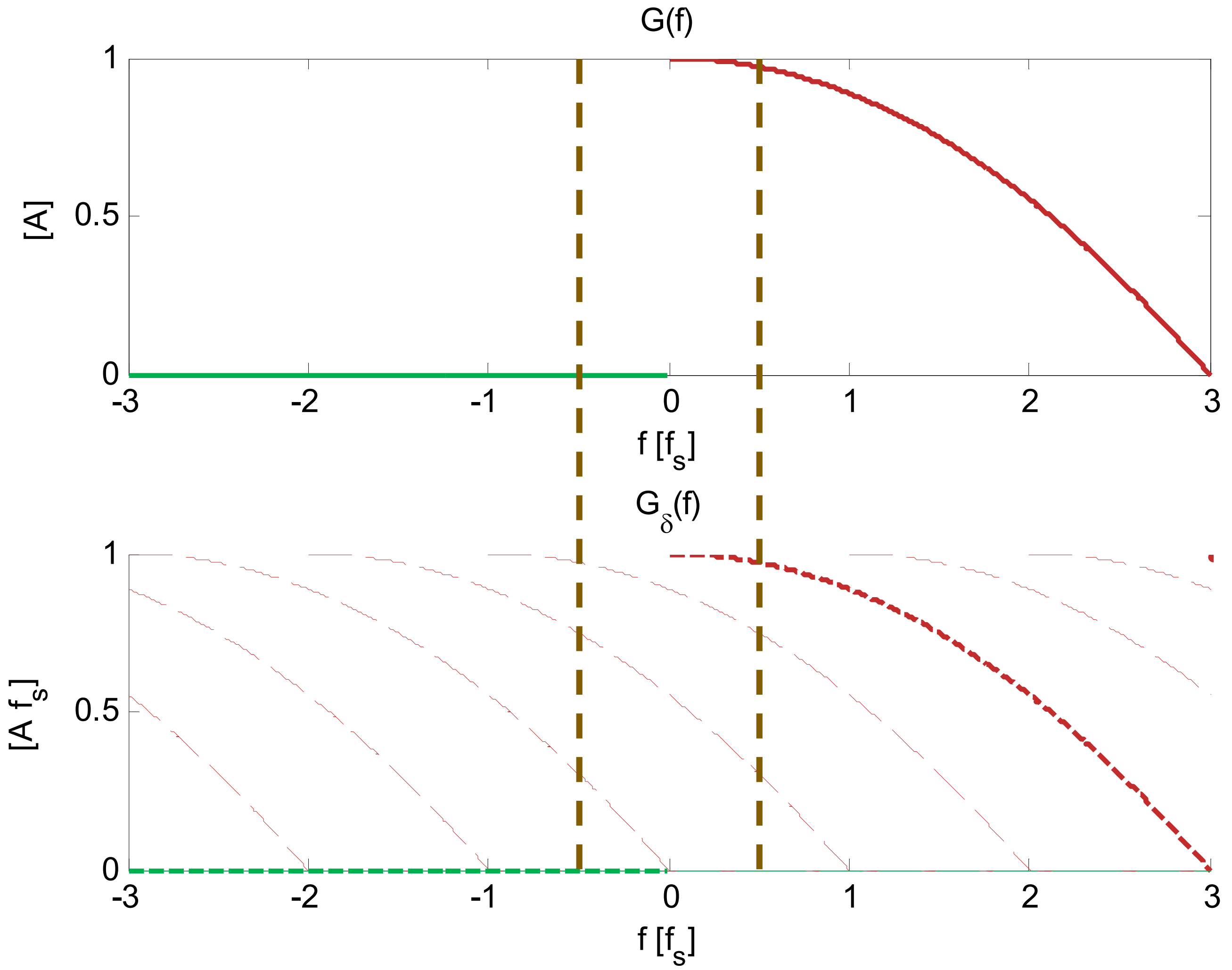


Ideal Sampling: Tunneling

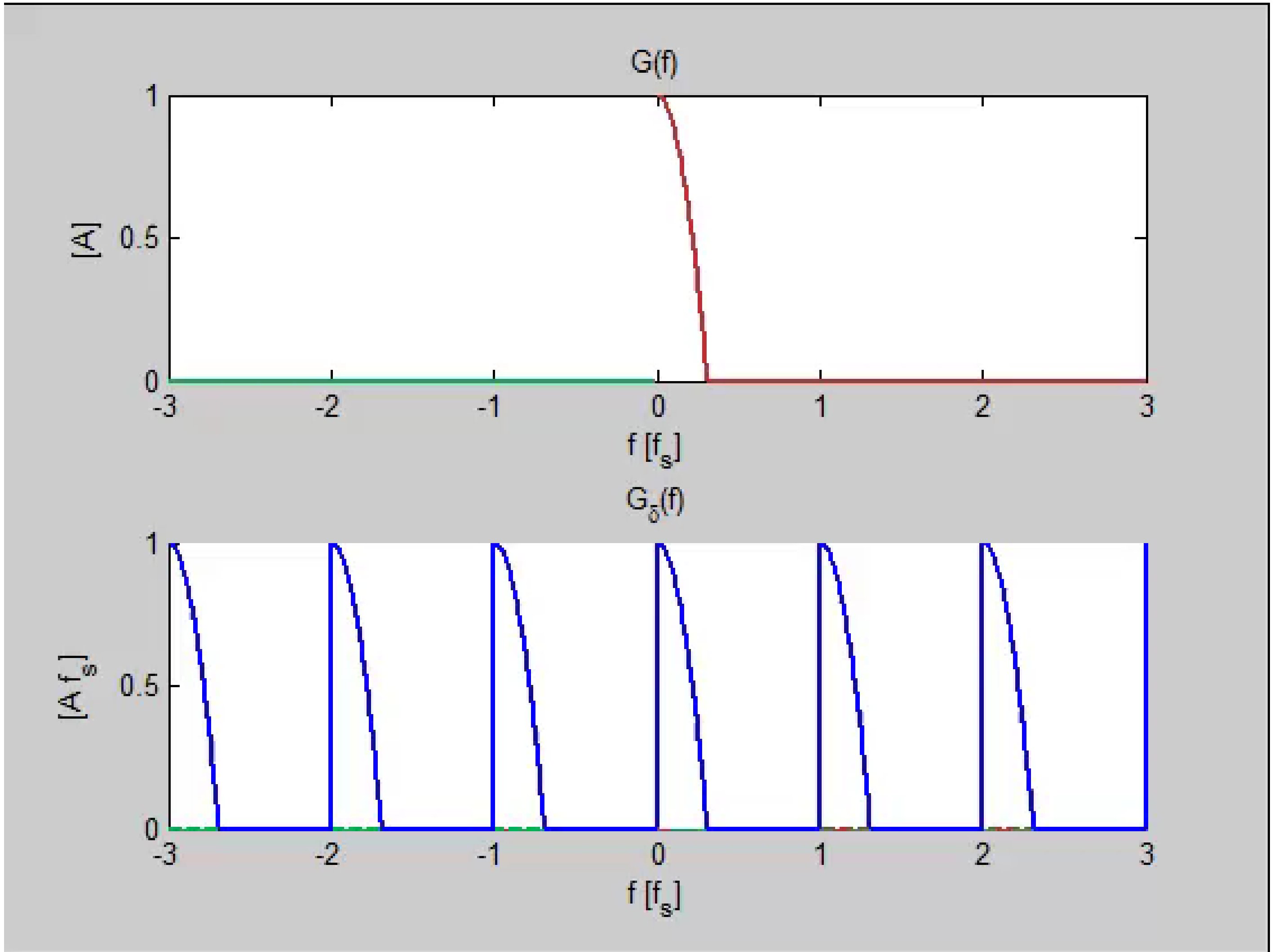


[Gdelta_demo7.m]

Ideal Sampling: Tunneling

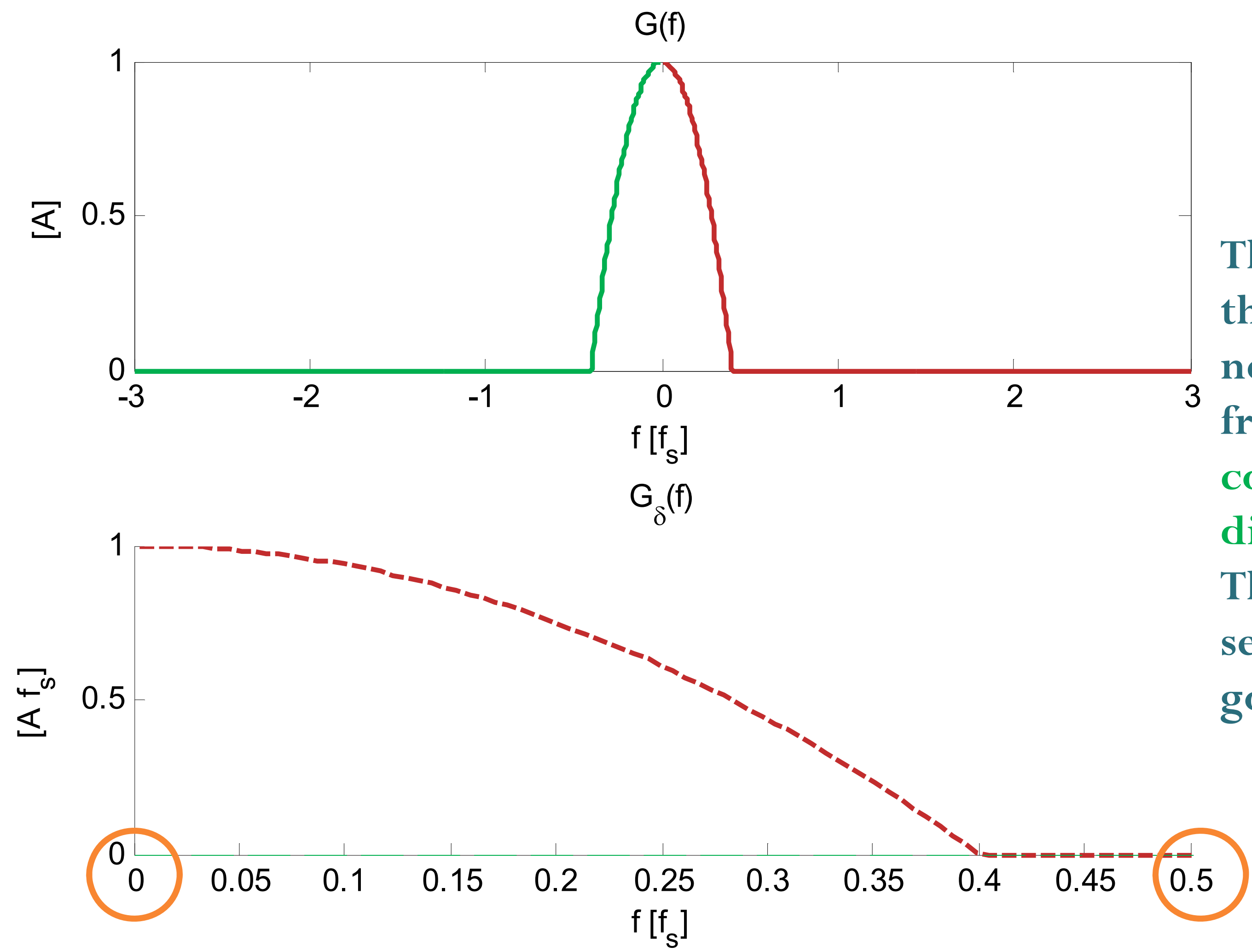


Ideal Sampling: Tunneling



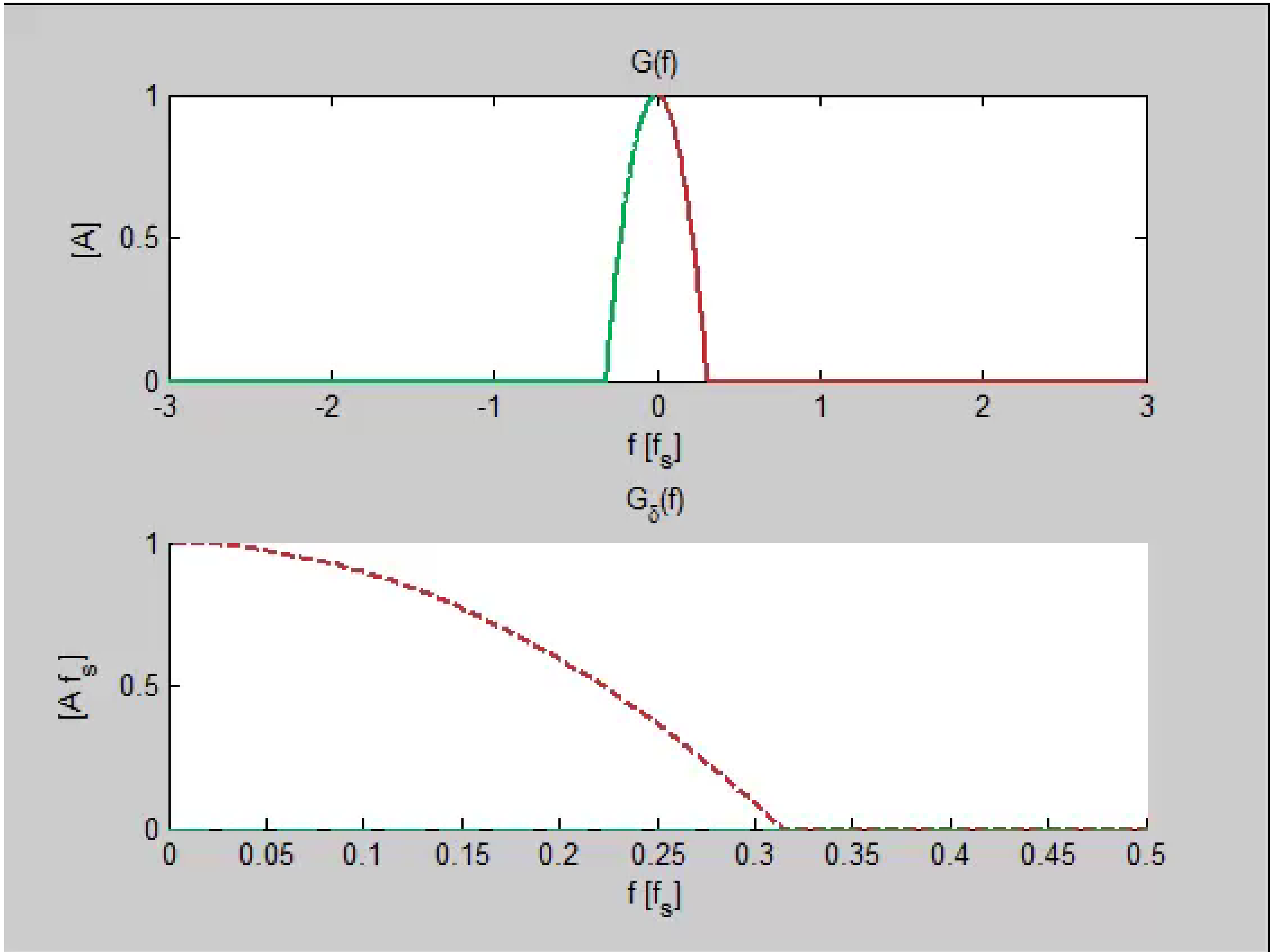
[Gdelta_demo8.m]

Ideal Sampling: Folding (a revisit)



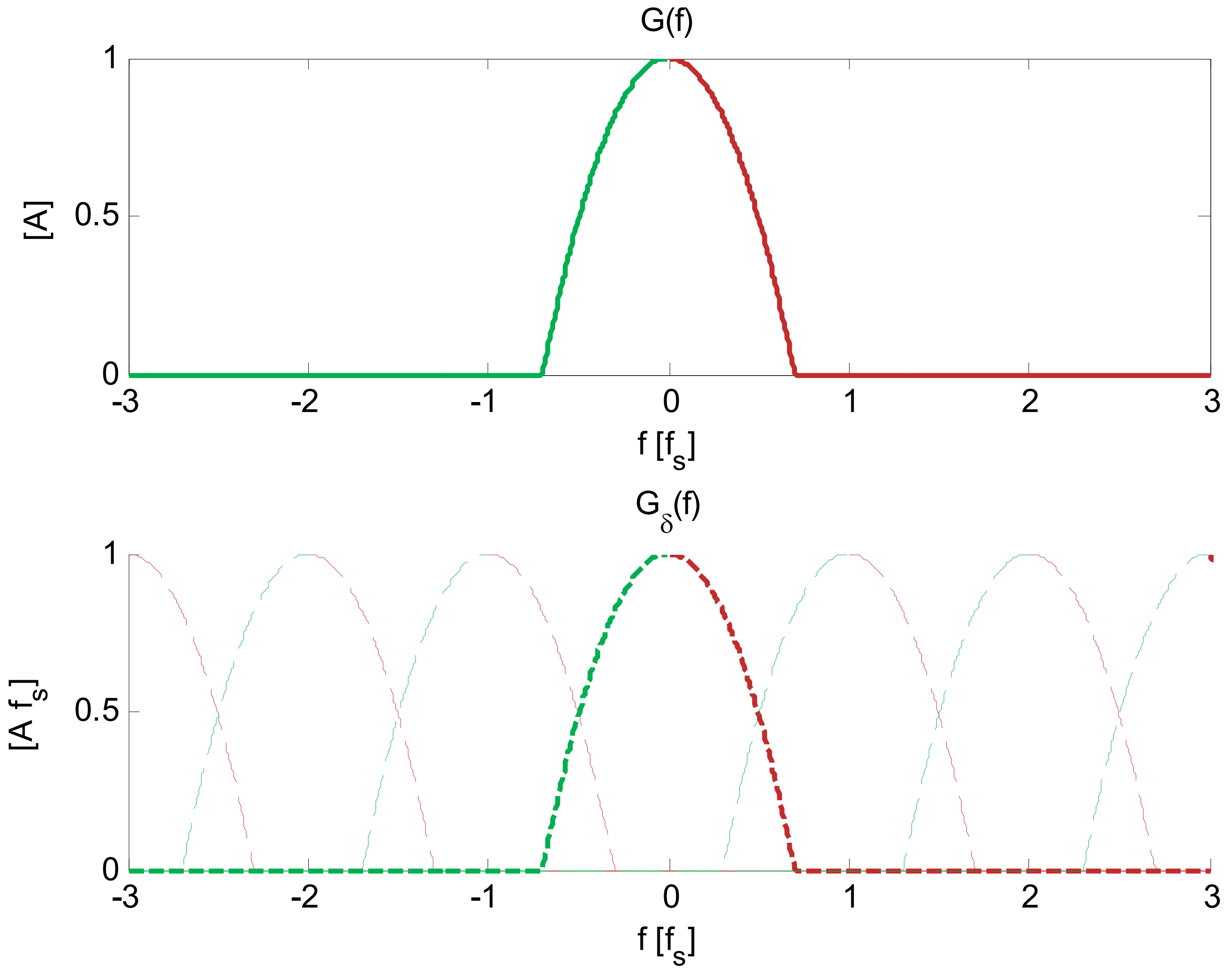
The part of G(f) that resides in the negative frequency is colored differently here. This allows us to see what really is going on.

Ideal Sampling: Folding (a revisit)

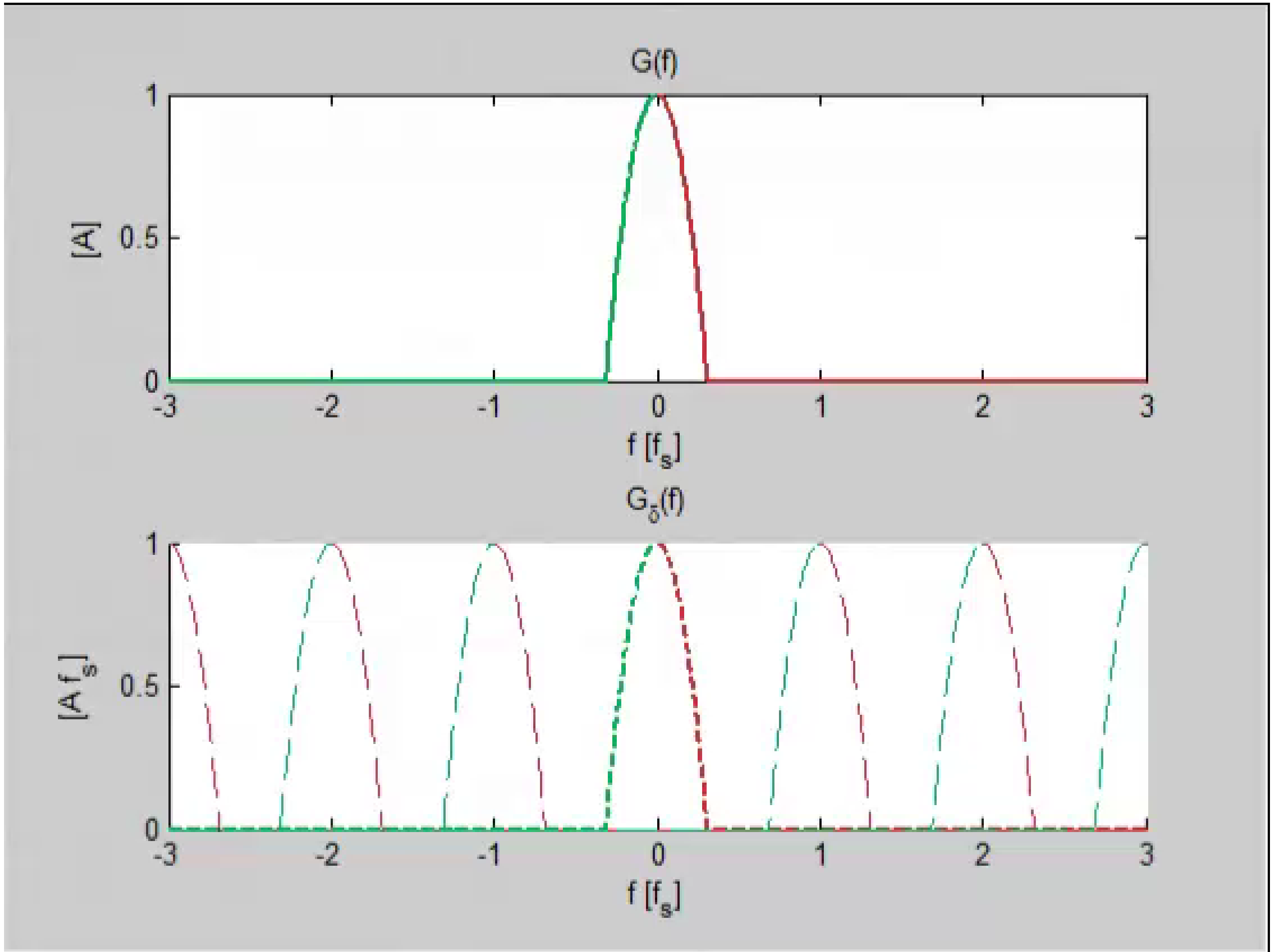


[Gdelta_demo9.m]

Ideal Sampling: Folding (a revisit)

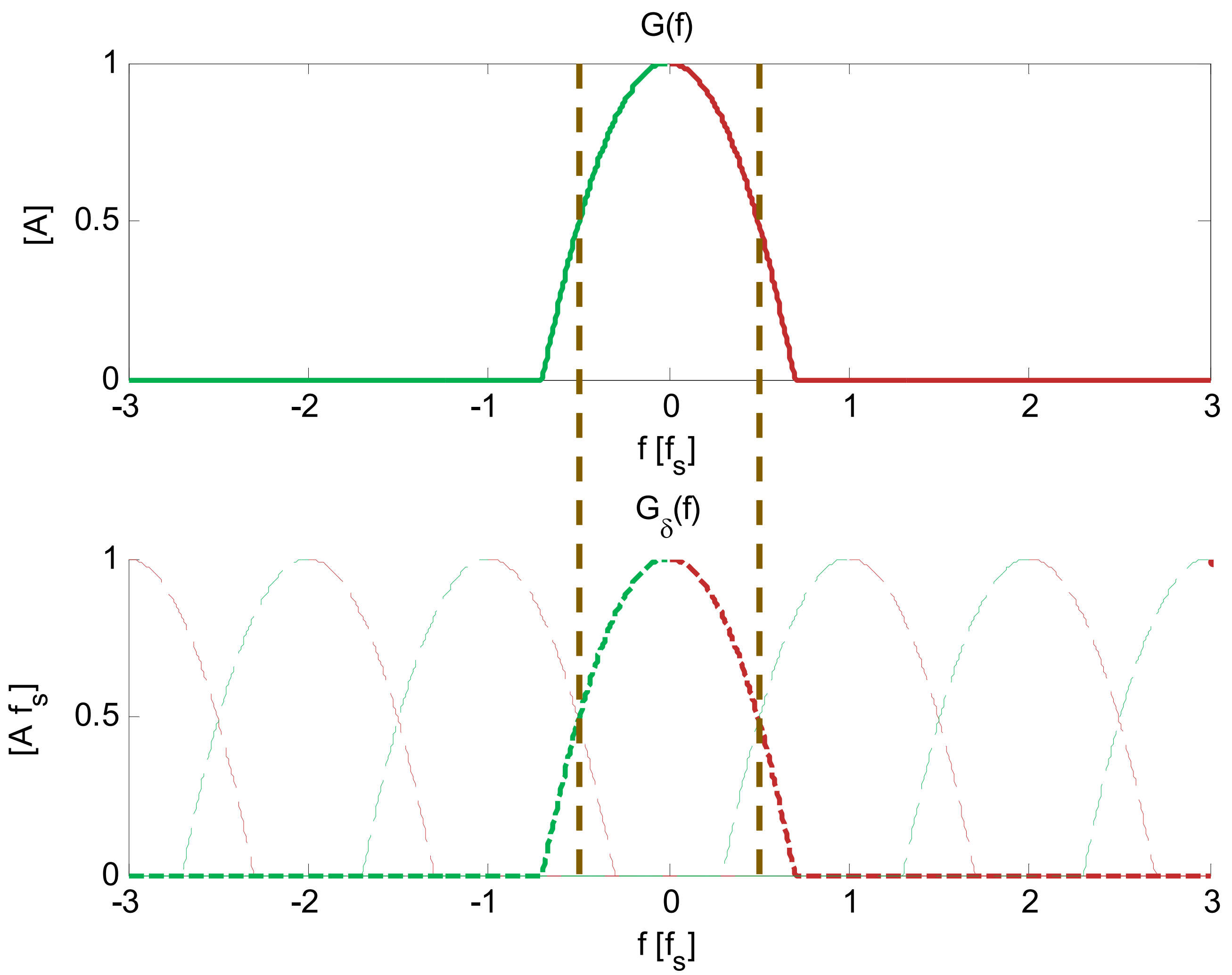


Ideal Sampling: Folding (a revisit)



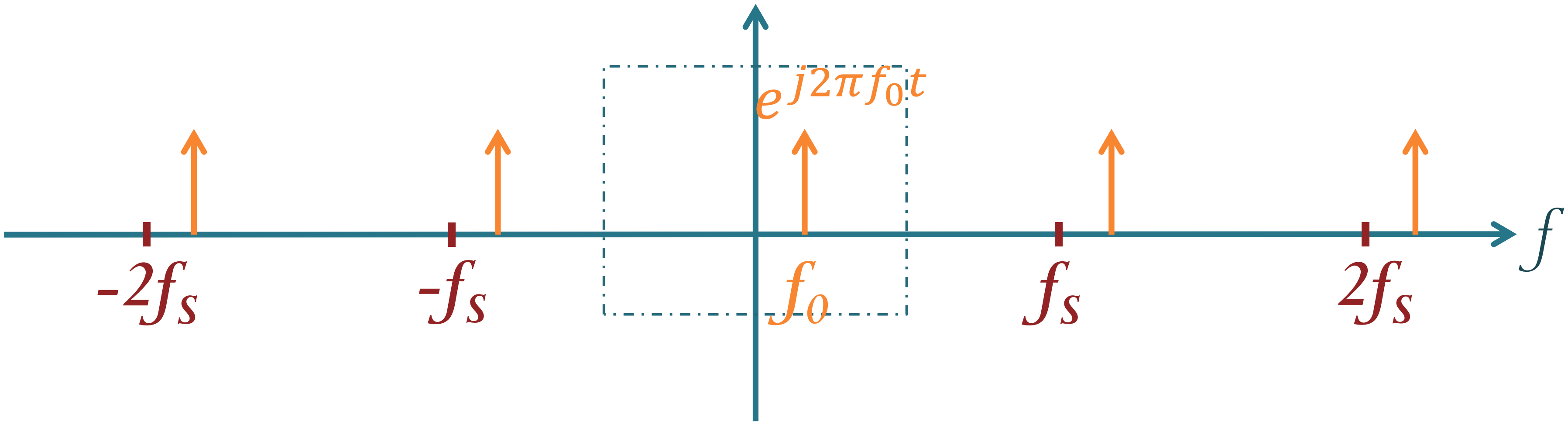
[Gdelta_demo10.m]

Ideal Sampling

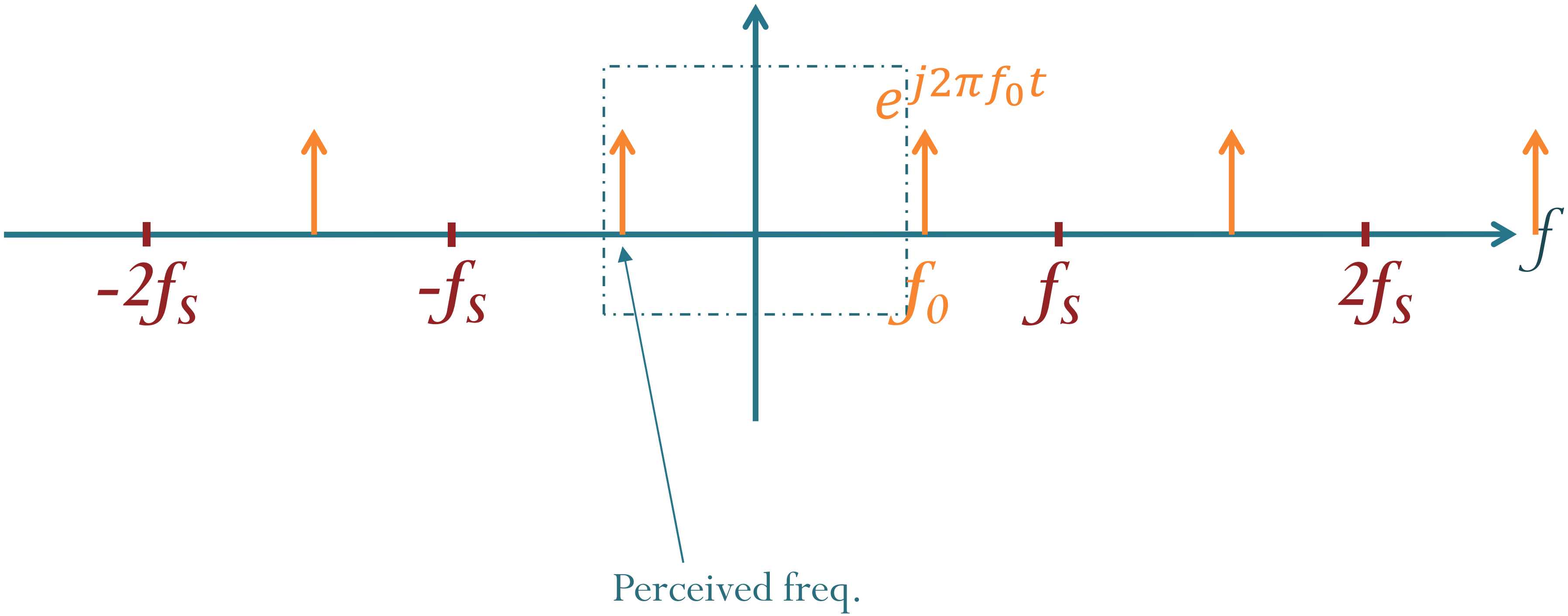


Complex exponential

Let's increase f_0



Complex exponential



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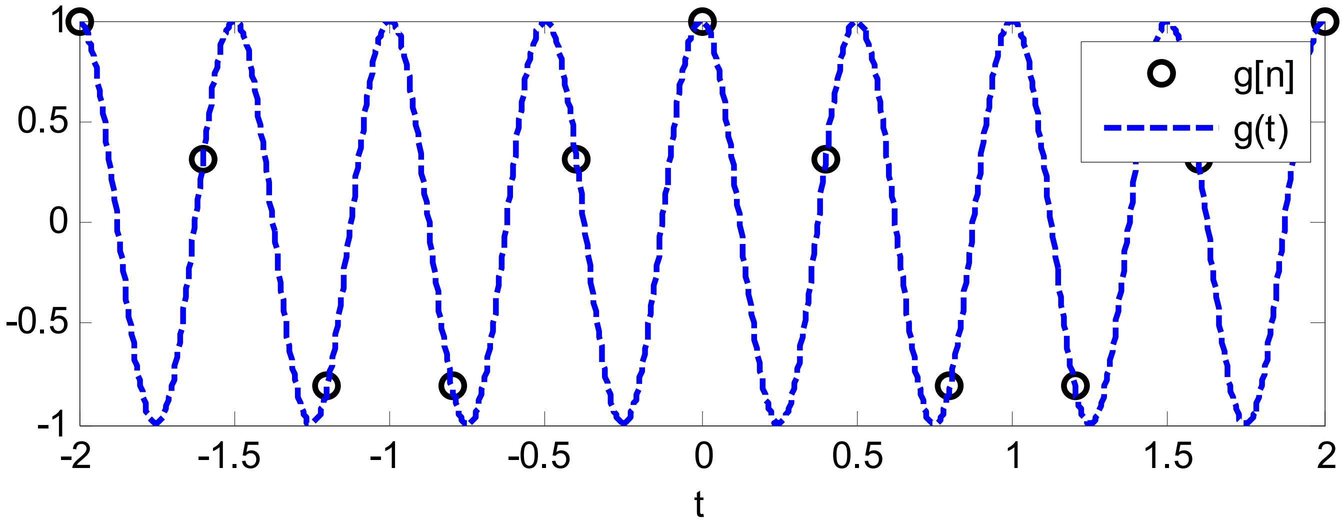
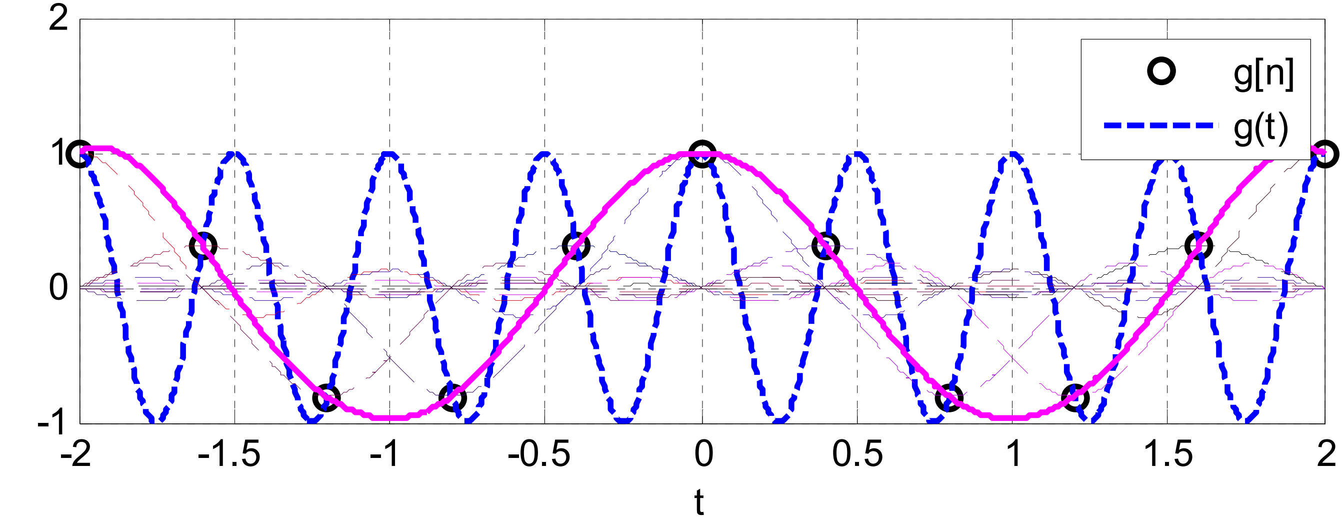
6.2 Reconstruction

Reconstruction of $\cos(2\pi(2)t)$

$T_s = 0.4$



Upper plot in Figure 34.

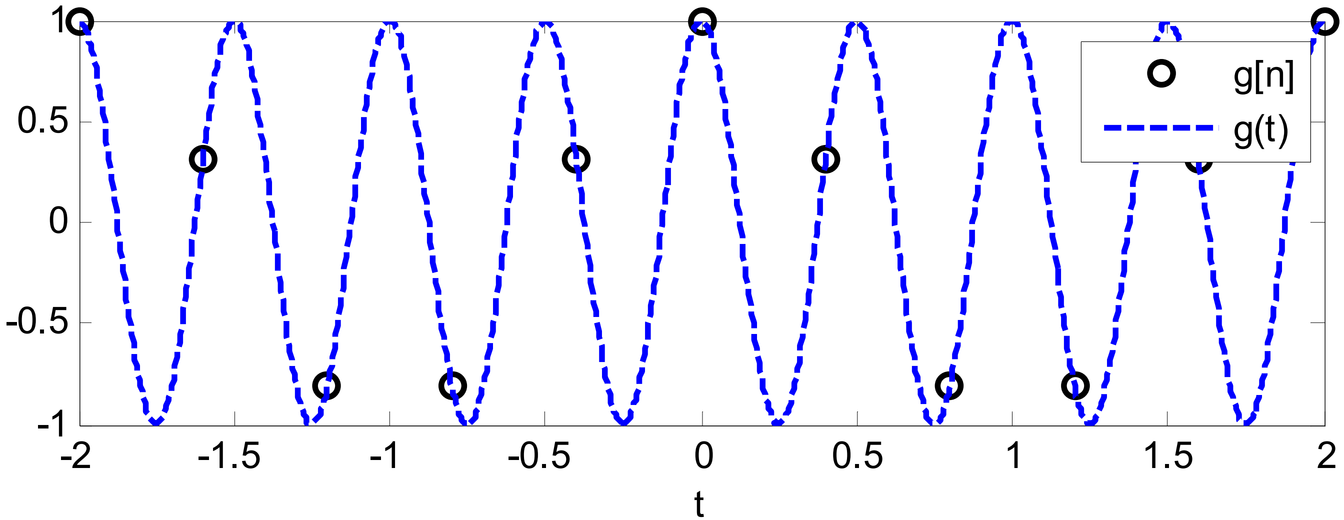


Reconstruction of $\cos(2\pi(2)t)$

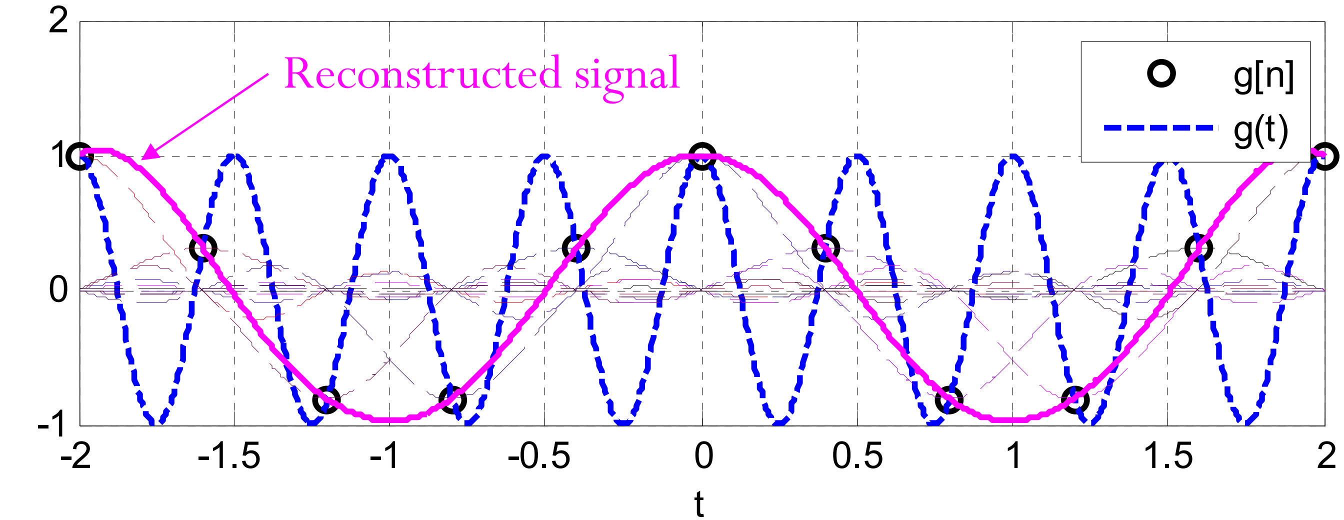
B = 2 Hz.

$T_s = 0.4$

$f_s = 1/0.4$
$= 2.5$ [Sa/s]

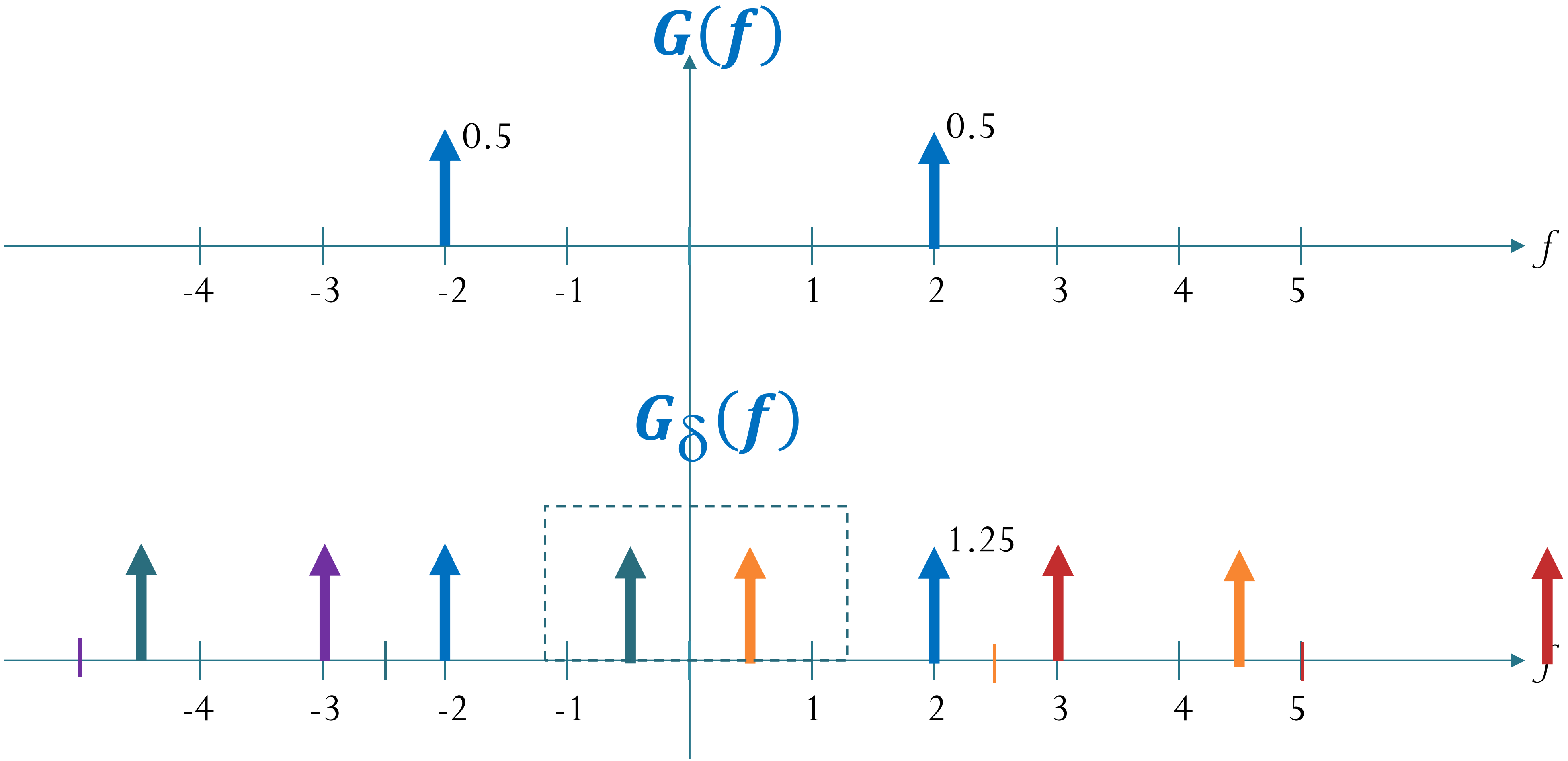


Upper plot in Figure 34.



$f_s < 2B \Rightarrow$ the **reconstructed signal** is different from the **original signal.**

Reconstruction of $\cos(2\pi(2)t)$

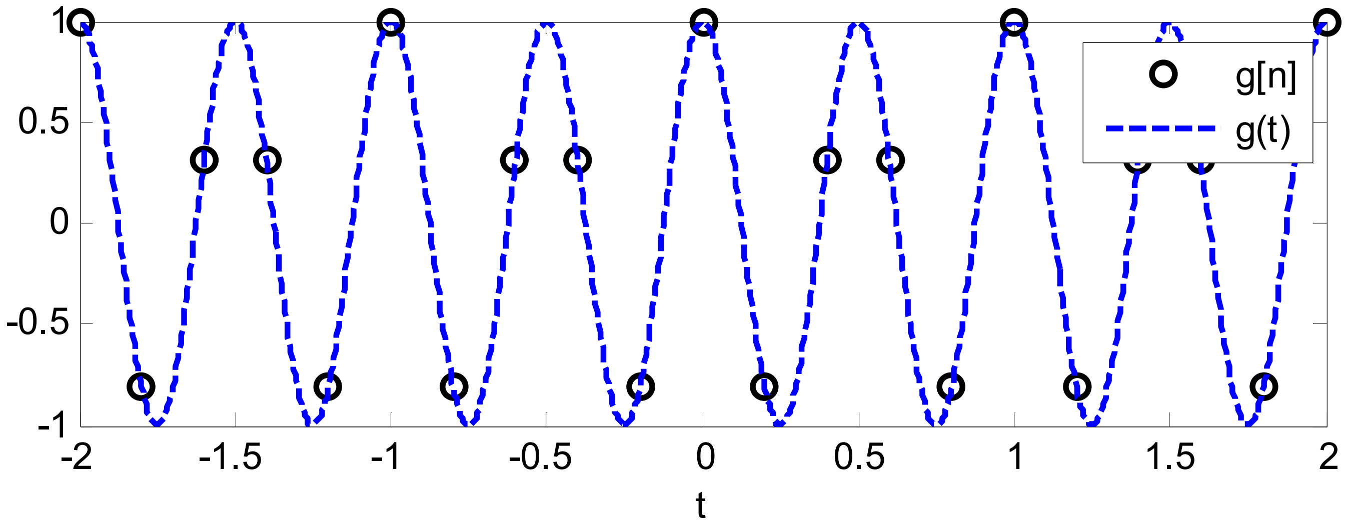


Reconstruction of $\cos(2\pi(2)t)$

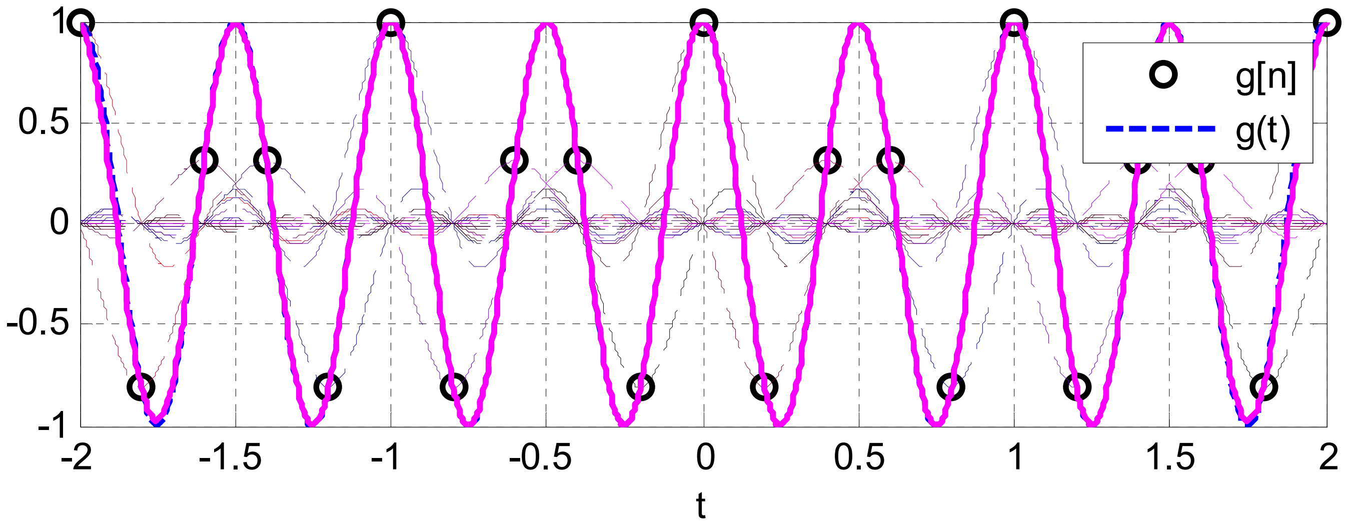
B = 2 Hz.

$T_s = 0.2$

$f_s = \frac{1}{0.2} = 5$ [Sa/s]



Lower plot in Figure 34.

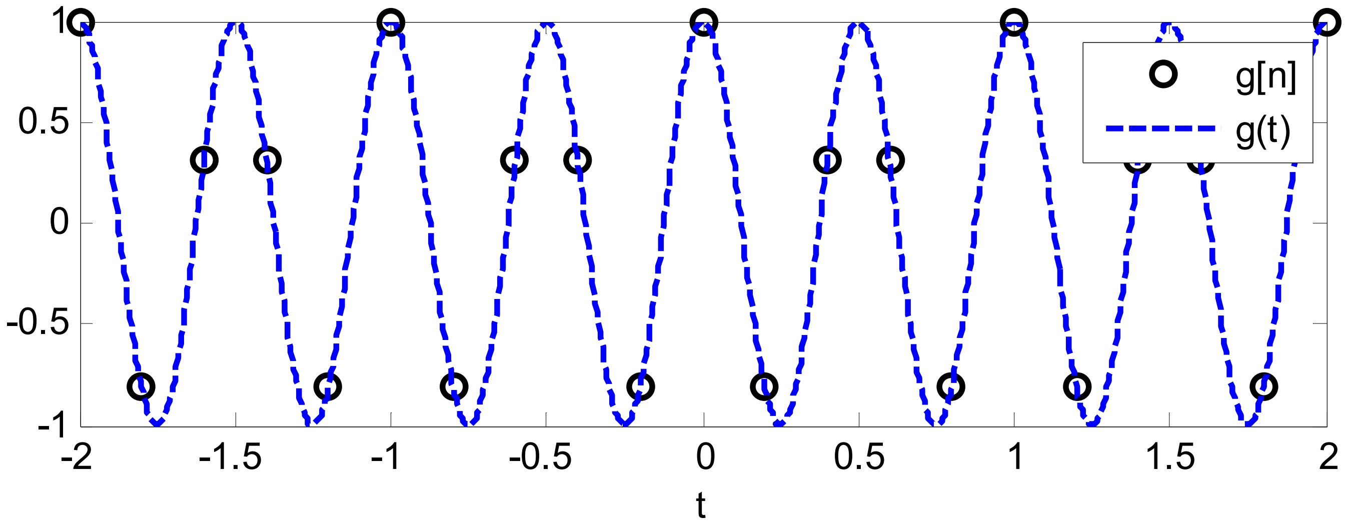


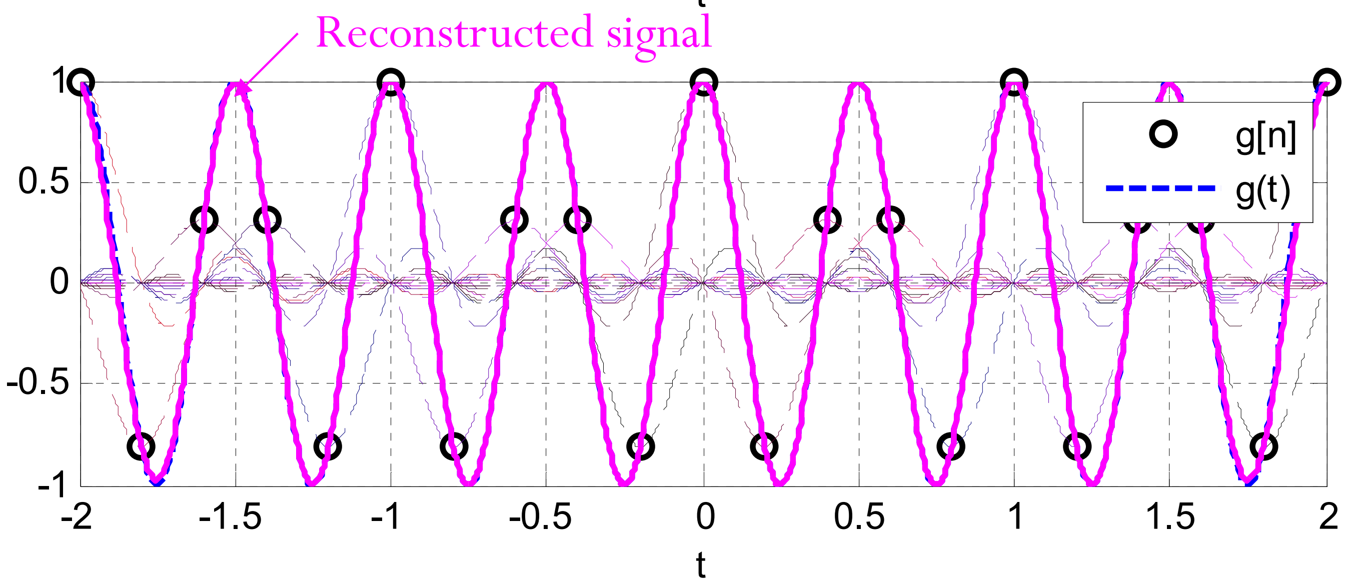
B = 2 Hz.

Reconstruction of $\cos(2\pi(2)t)$

$T_s = 0.2$

$f_s = \frac{1}{0.2} = 5$ [Sa/s]





Lower plot in Figure 34.

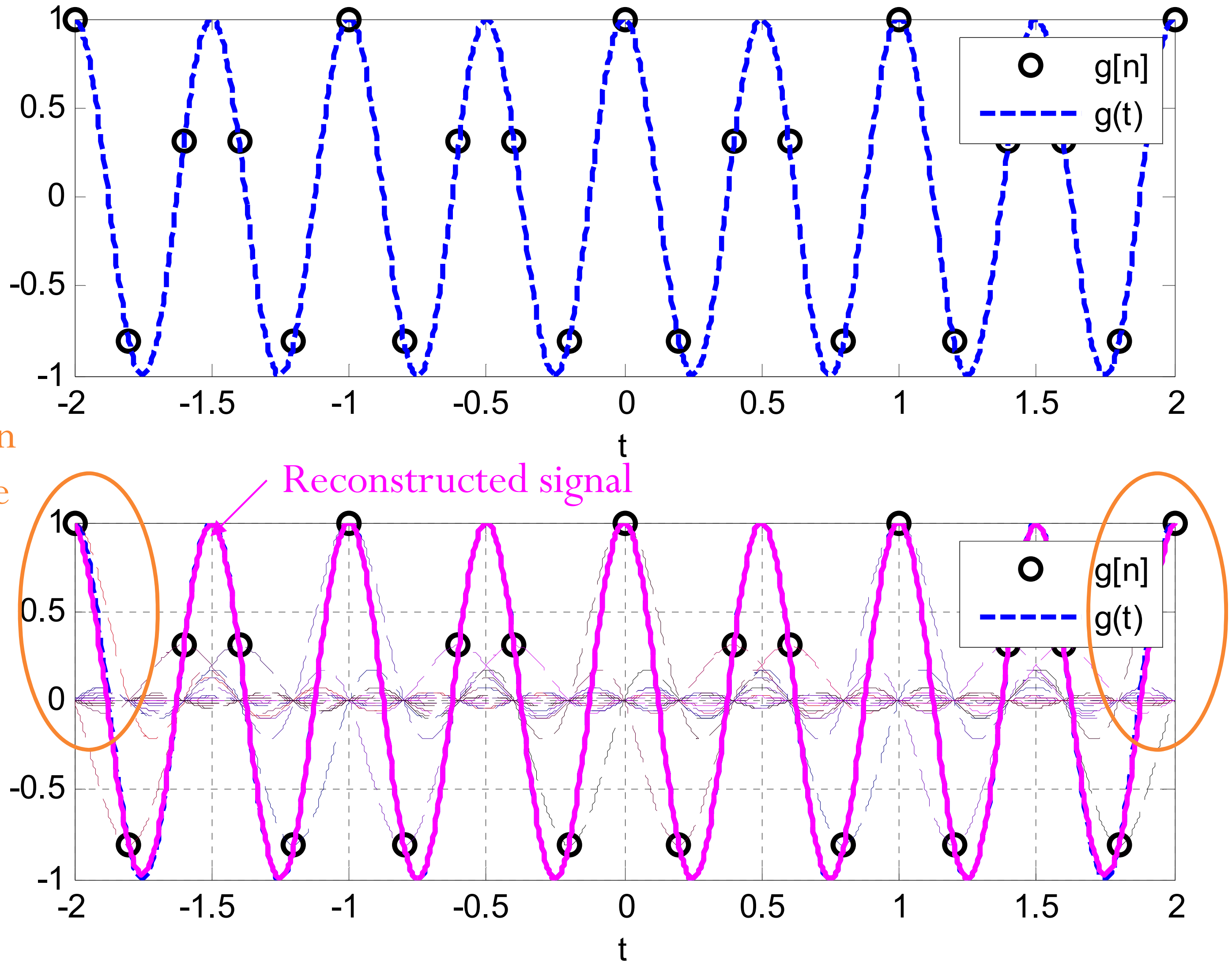
$f_s > 2B \Rightarrow$ the **reconstructed signal** is "the same" as the **original signal.**

Reconstruction of $\cos(2\pi(2)t)$

B = 2 Hz.

$T_s = 0.2$

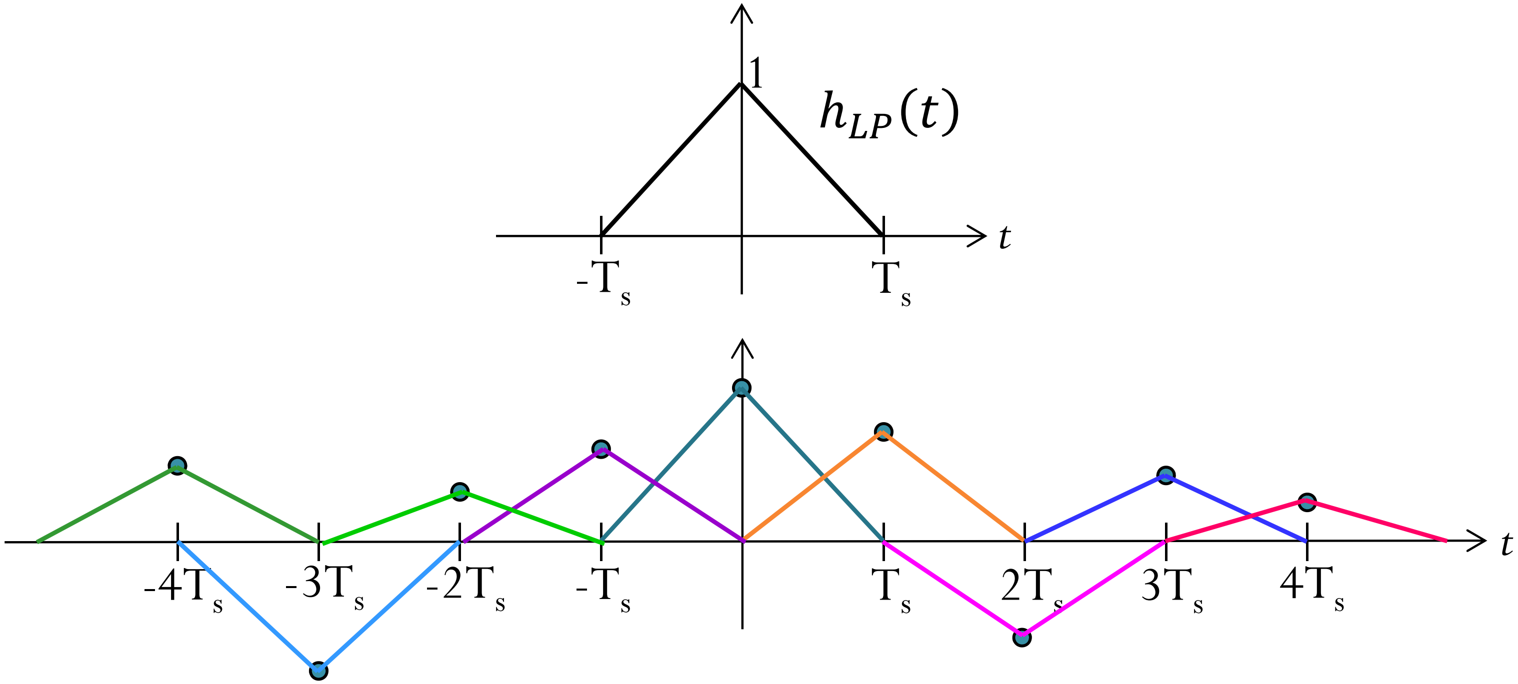
$f_s = \frac{1}{0.2} = 5$ [Sa/s]



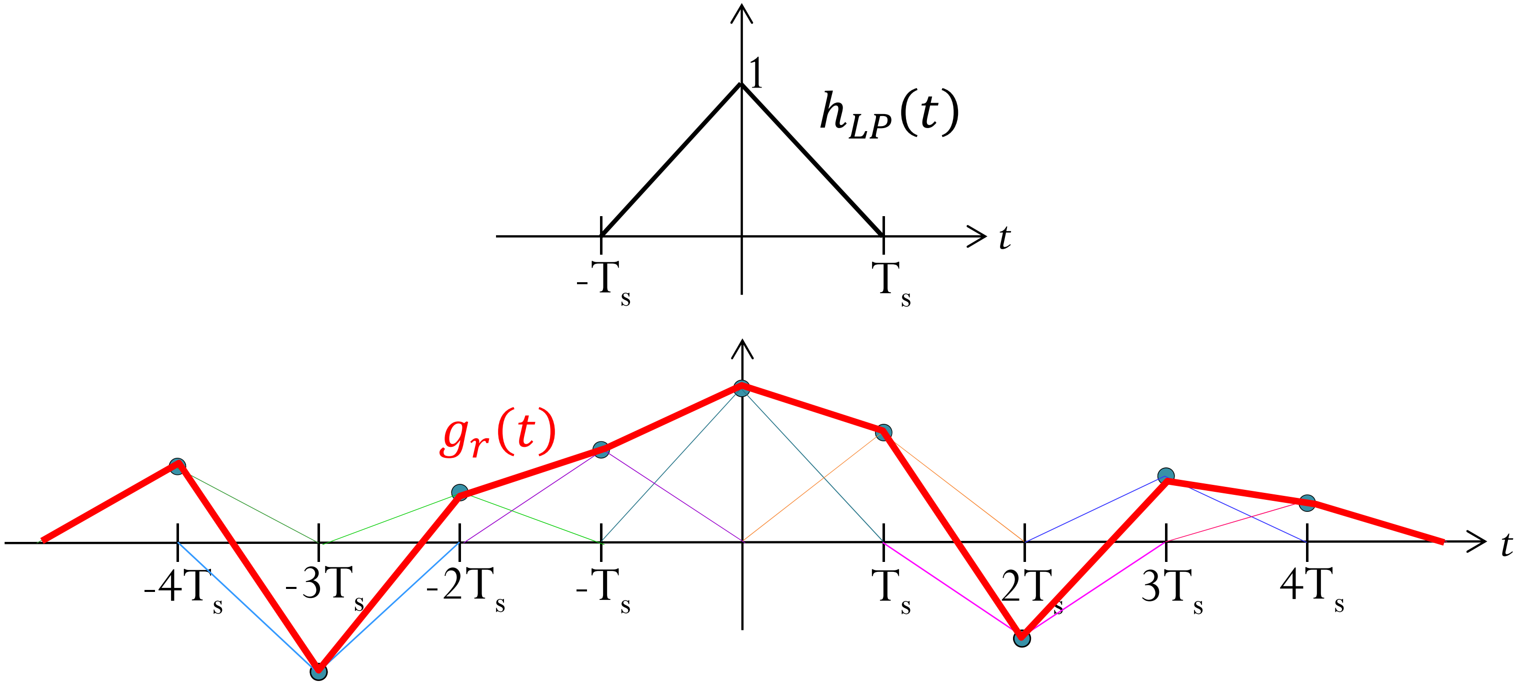
Some reconstruction error is visible at the boundaries because we did not use g[n] for n beyond ±2 in the reconstruction here.

$f_s > 2B \Rightarrow$ the reconstructed signal is "the same" as the original signal.

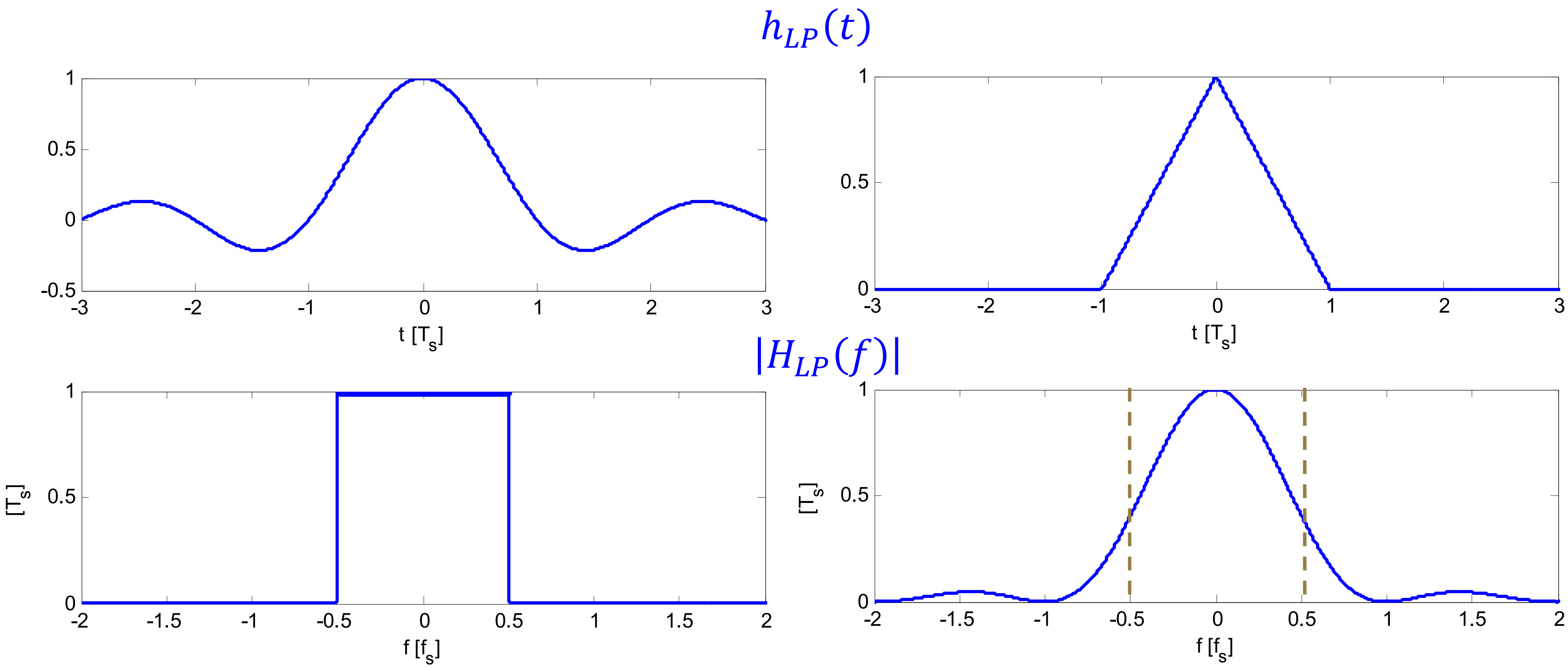
Triangular (linear) interpolation



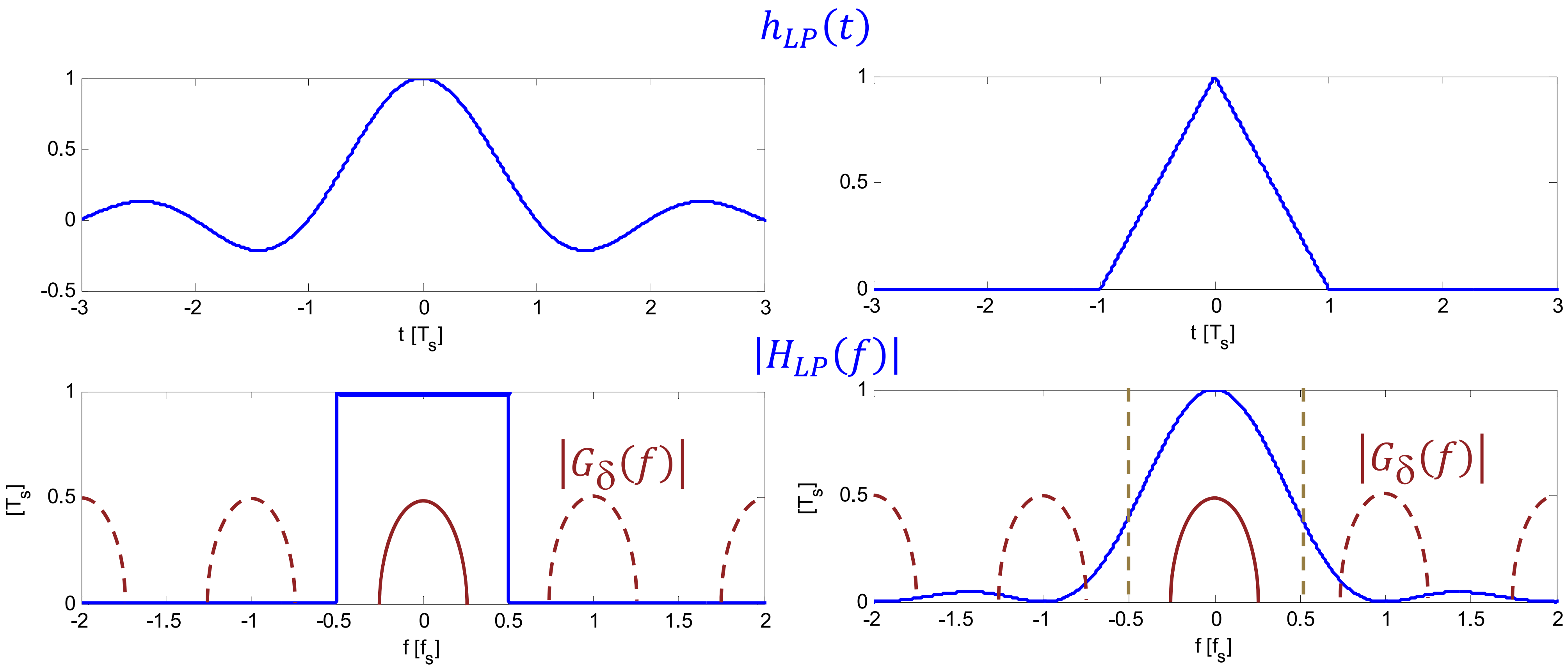
Triangular (linear) interpolation



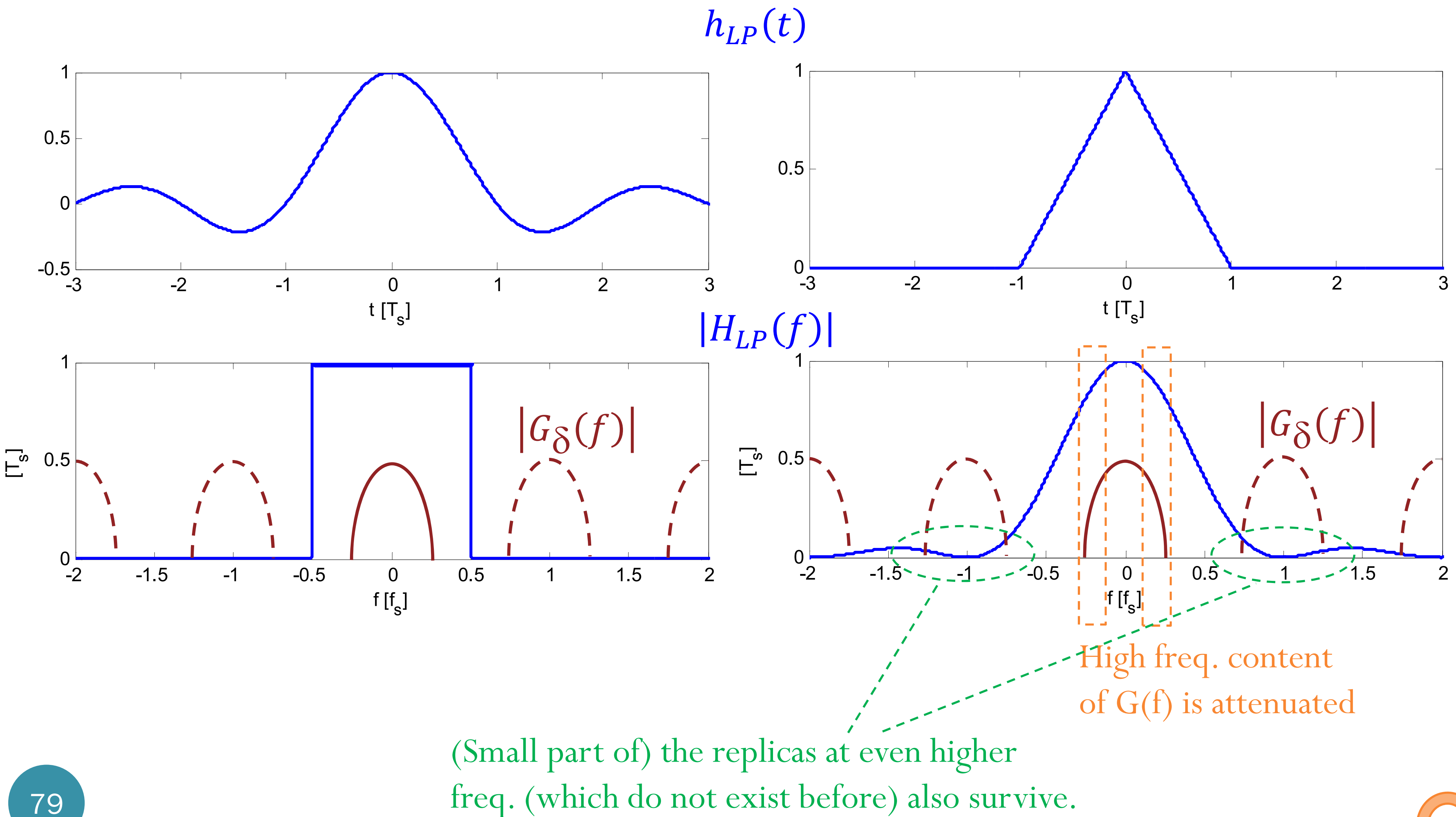
sinc vs. triangular interpolation



sinc vs. triangular interpolation



sinc vs. triangular interpolation



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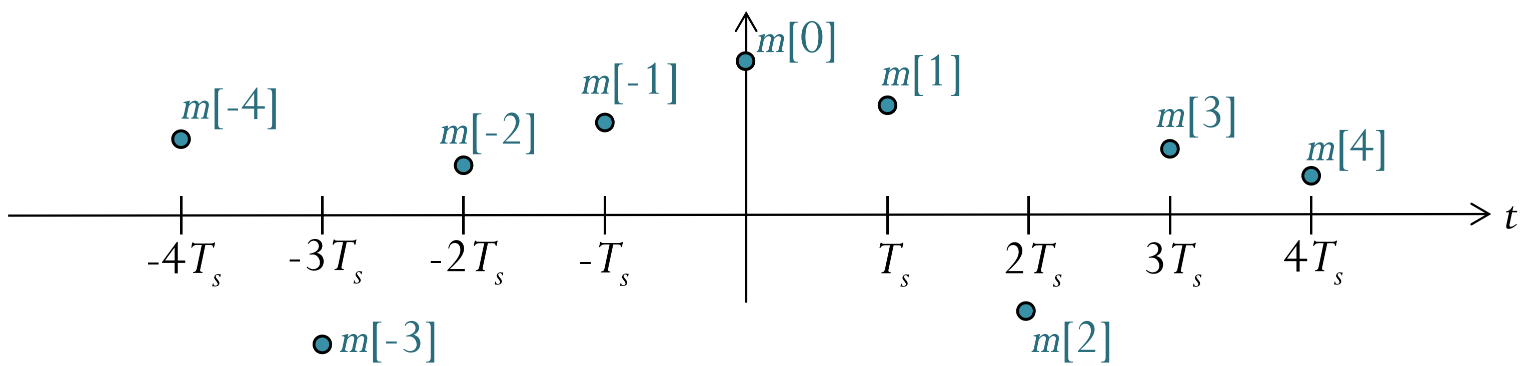
6.3 Analog Pulse Modulation

PAM: Pulse Amplitude Modulation

Start with a sequence of symbols (numbers).



Where does this sequence come from?

- Sampling of a continuous-time signal
- Naturally discrete-time signal

Naturally digital information

- Text is commonly encoded using ASCII, and MATLAB automatically represents any string file as a list of ASCII numbers.

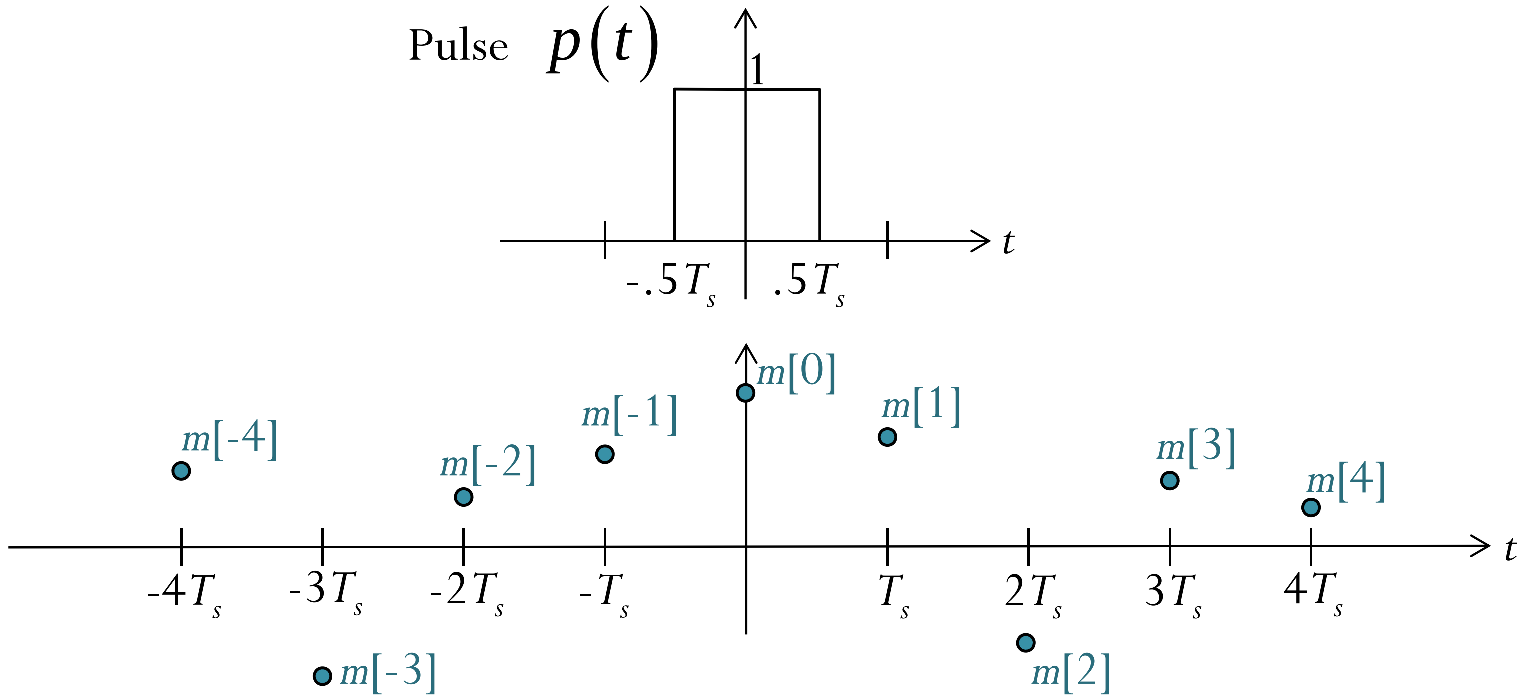
```
>> str='I love ECS332';
>> real(str)

ans =

    73    32   108   111   118   101    32    69    67    83    51    51    50

>> dec2base(str,2)

ans =

1001001
0100000
1101100
1101111
1110110
1100101
0100000
1000101
1000011
1010011
0110011
0110011
0110010
```

text string

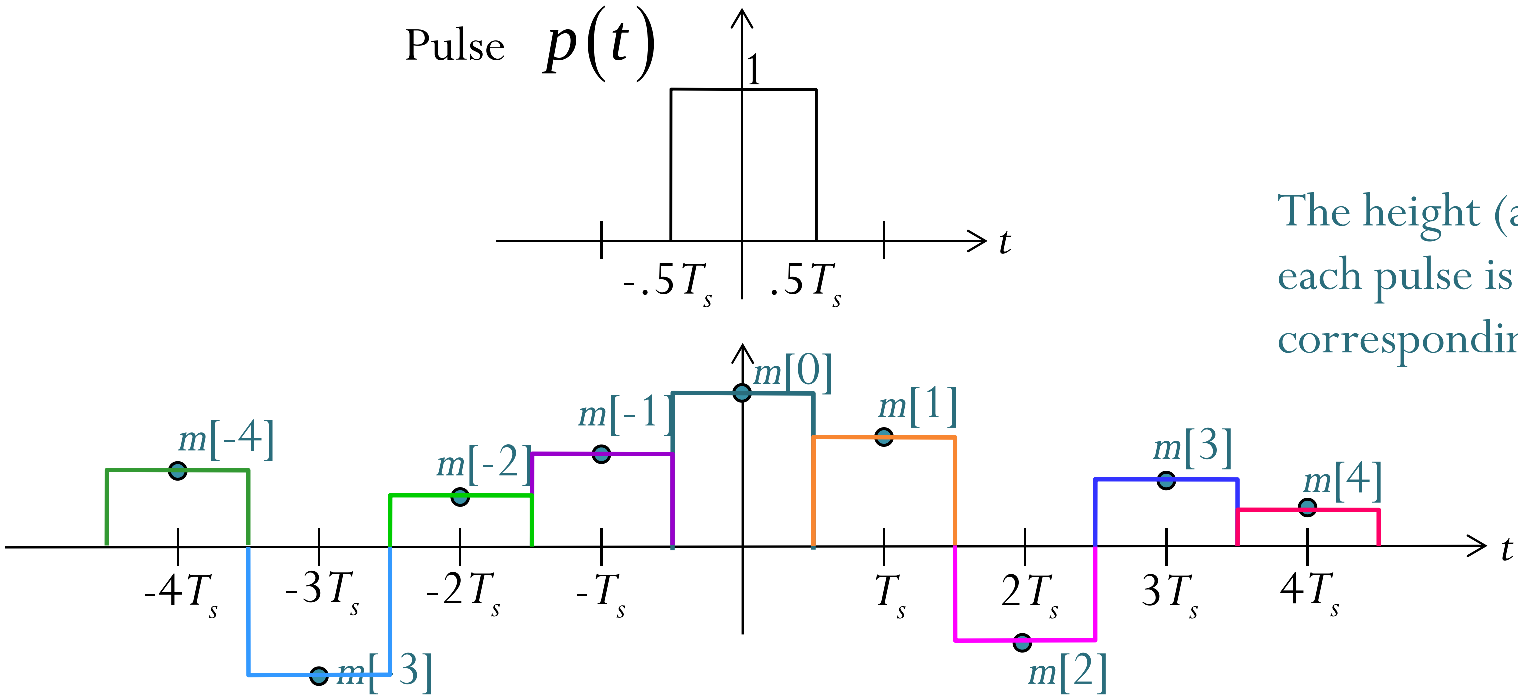
(decimal) ASCII representation of the text string

binary (base 2) representation of the decimal numbers

PAM: Pulse Amplitude Modulation

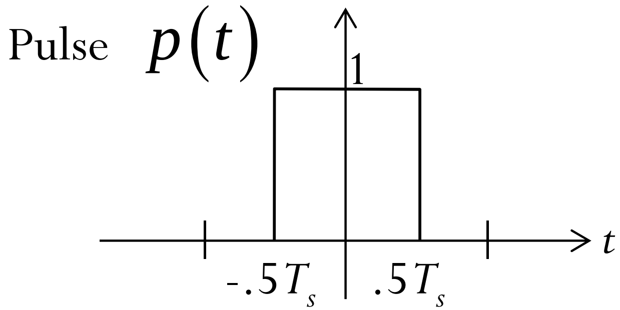


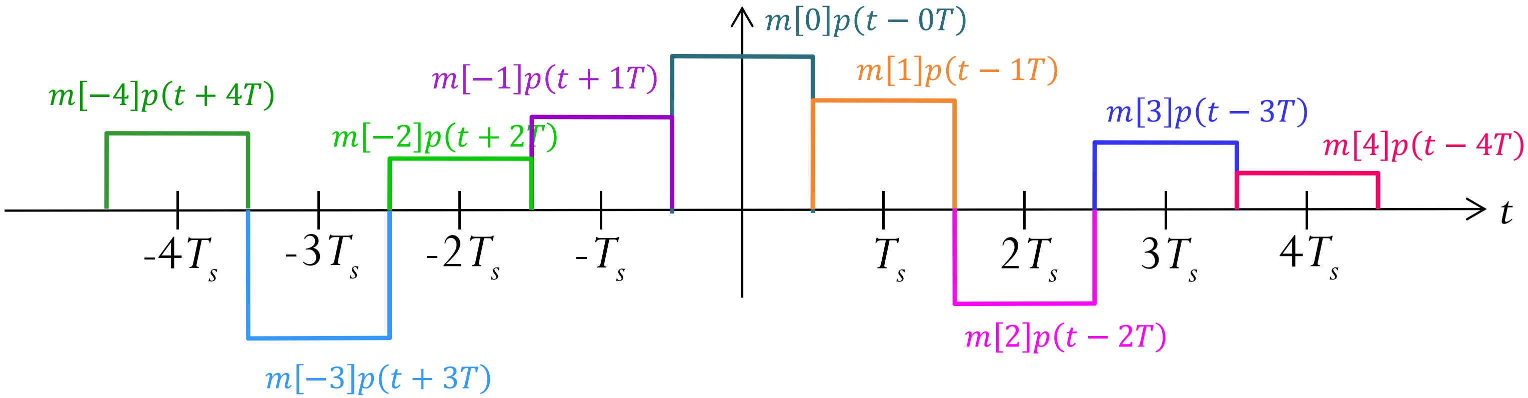
PAM: Pulse Amplitude Modulation



The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

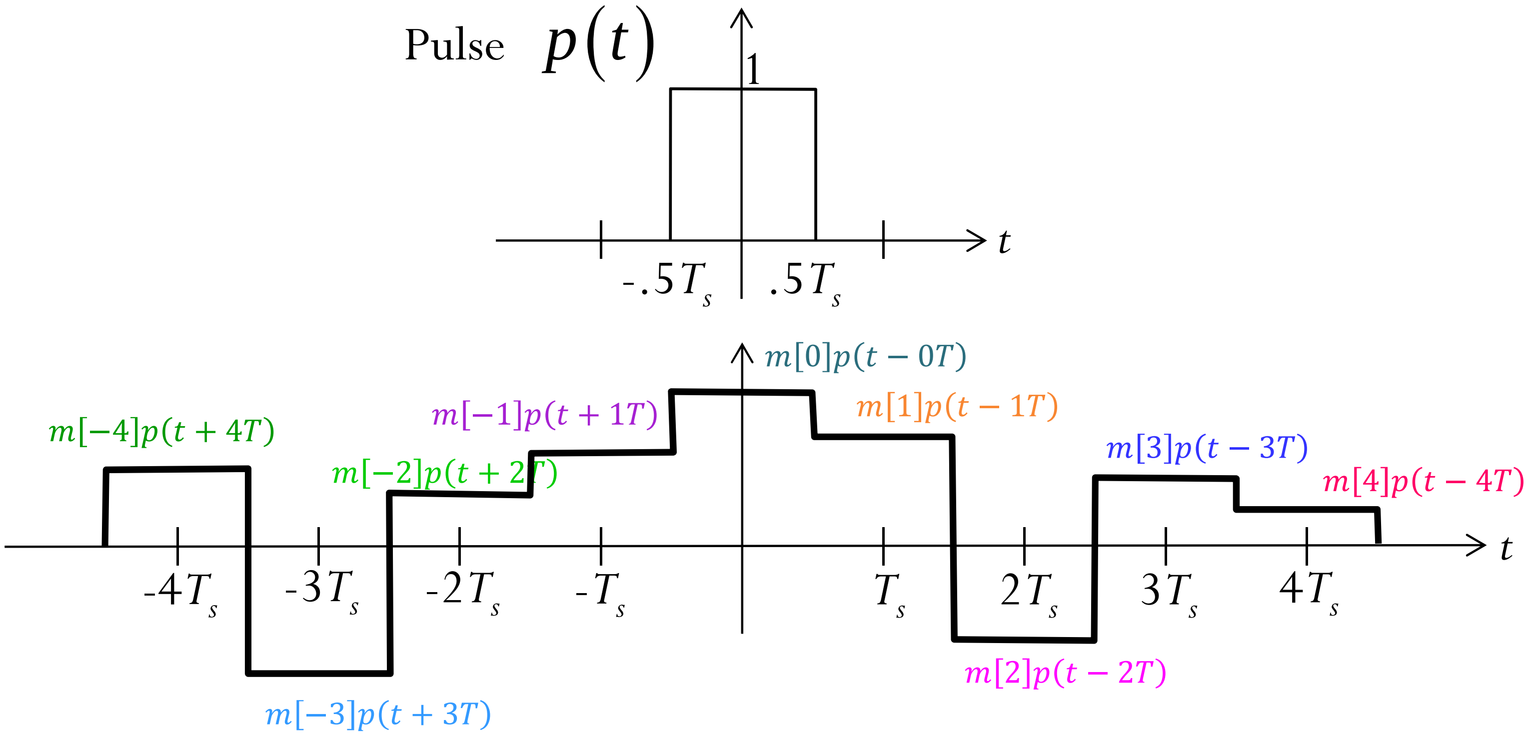
PAM: Pulse Amplitude Modulation





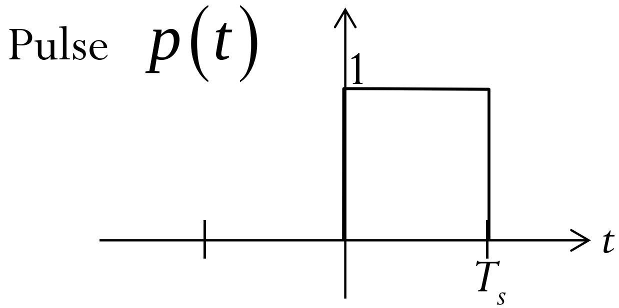
$$x_{\text{PAM}}(t)=\sum_{n=-\infty}^{\infty}m[n]\,p(t-nT)$$

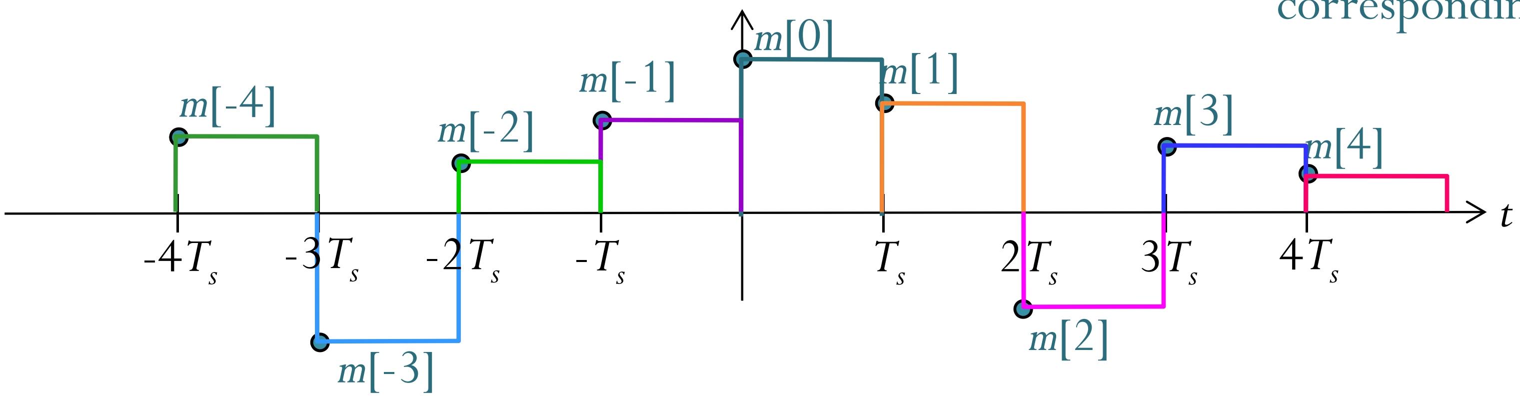
PAM: Pulse Amplitude Modulation



$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t-nT)$$

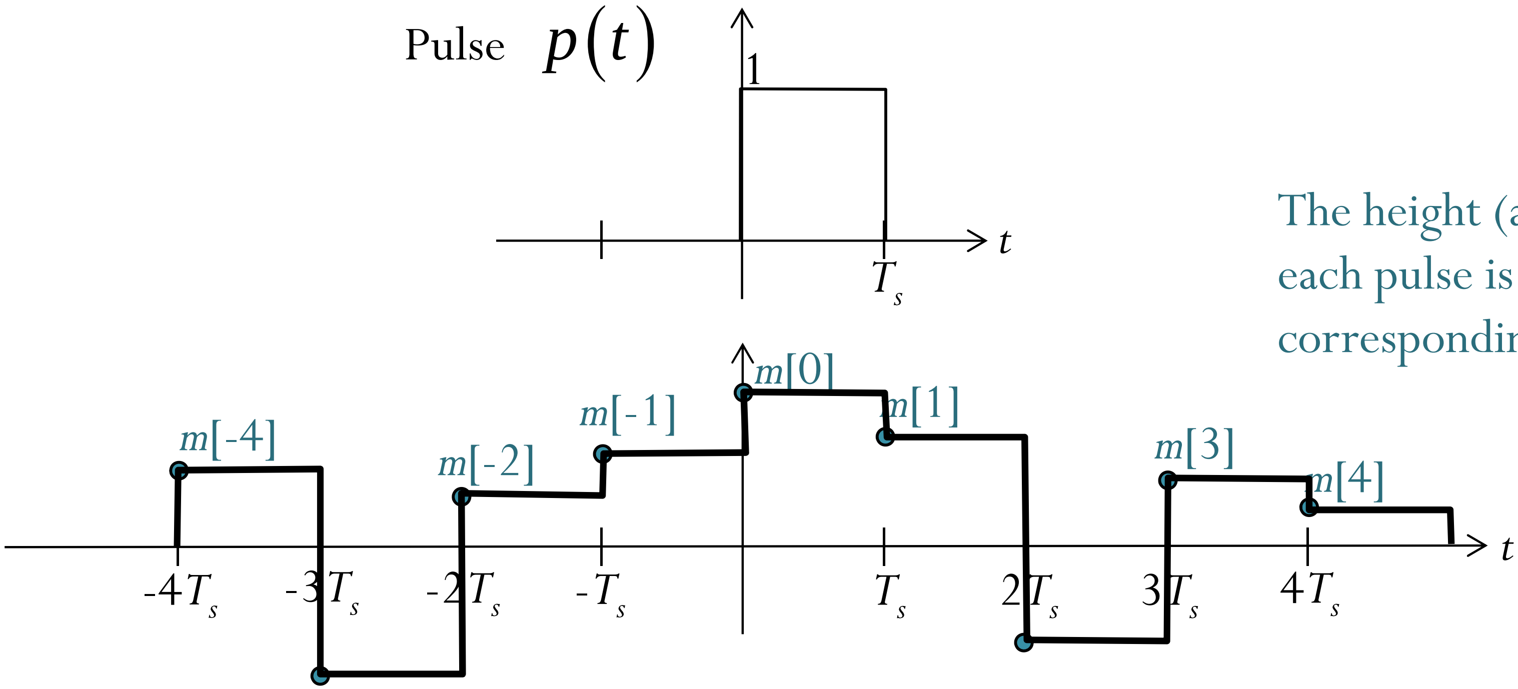
PAM: Pulse Amplitude Modulation





The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

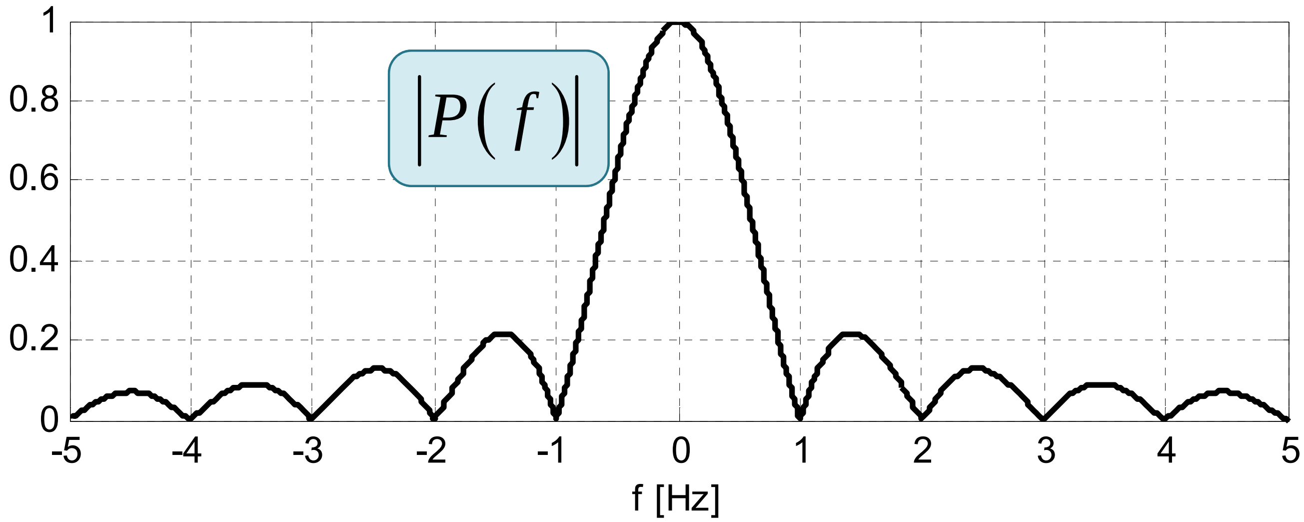
PAM: Pulse Amplitude Modulation



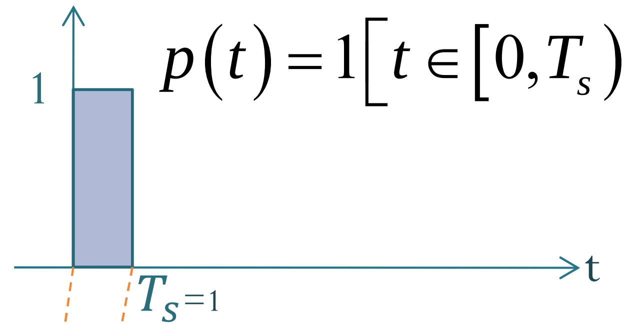
The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

$$x_{\mathrm{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t - nT)$$

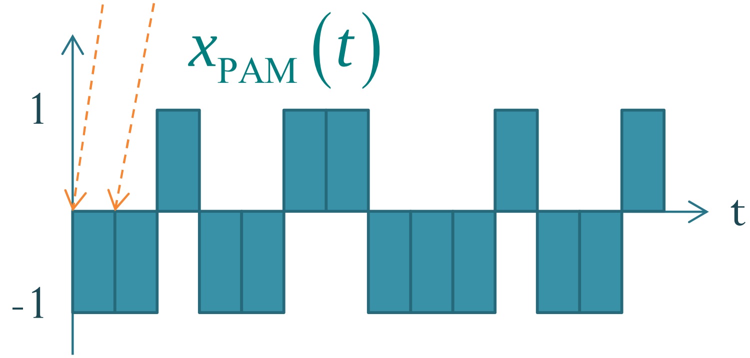
$X_{\mathrm{PAM}}(f)$ (1/4)



$$p(t) = 1\left[t \in [0, T_s)\right]$$



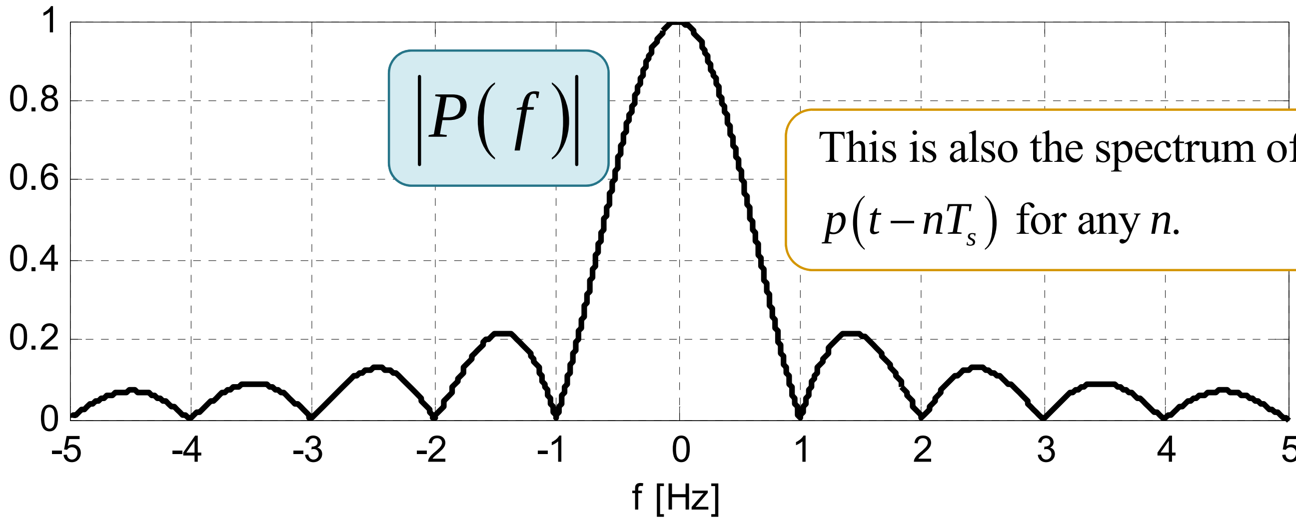
m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



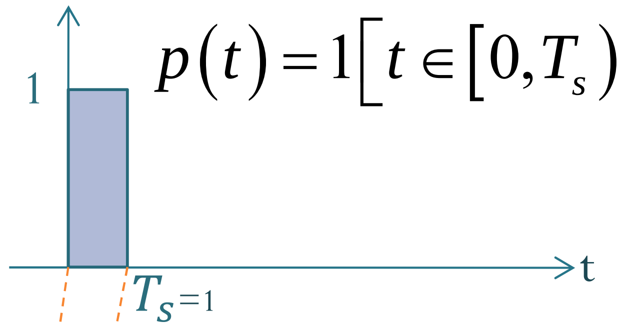
$$x_{\mathrm{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

Can you sketch the spectrum of $s(t)$?

$X_{\text{PAM}}(f)$ (2/4)

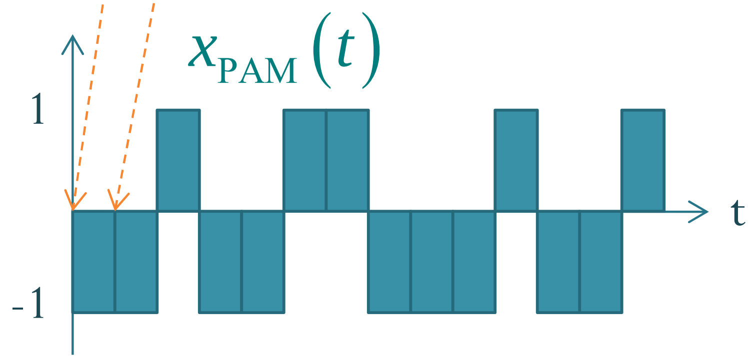


This is also the spectrum of $p(t-nT_s)$ for any n.



$$p(t) = 1\left[t \in [0, T_s)\right]$$

m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

Does this mean $|X_{\text{PAM}}(f)|$ will simply be a sum of $|P(f)|$ and therefore its shape will be similar to $|P(f)|$?

Important Properties of $\mathcal{F}$

$$\{x*y\}(t)=\int_{-\infty}^{\infty}x(\mu)y(t-\mu)d\mu=\int_{-\infty}^{\infty}x(t-\mu)y(\mu)d\mu$$

Convolution Properties:

$$x*y \underset{}{\overset{\mathcal{F}}{\rightleftharpoons}} X\times Y$$

$$x\times y \overset{\mathcal{F}}{\rightleftharpoons} X*Y$$

Note that the magnitude of this is simply $|G(f)|$

Shifting Properties:

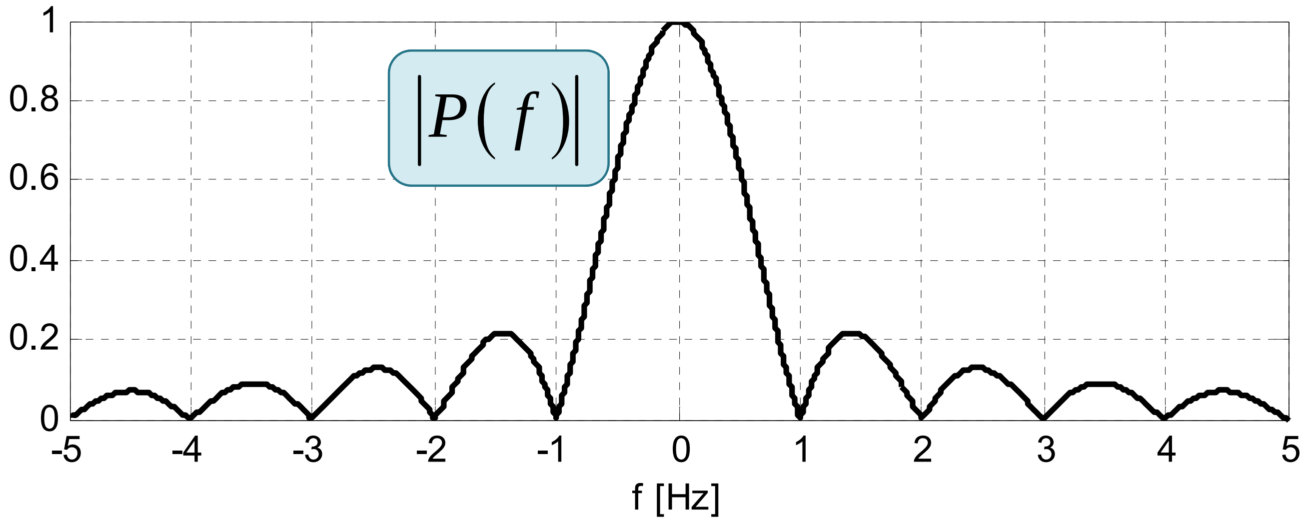
$$g(t-t_0) \overset{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0}G(f)$$

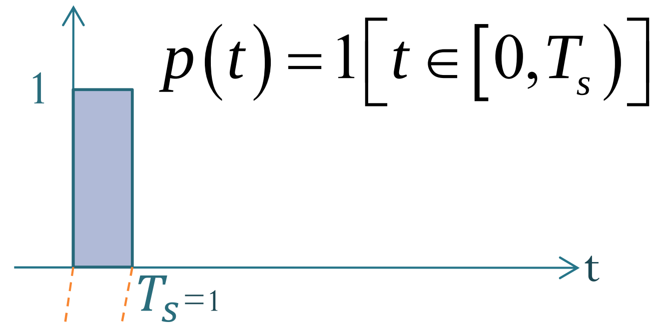
$$e^{j2\pi f_0 t}g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f-f_0)$$

Modulation:

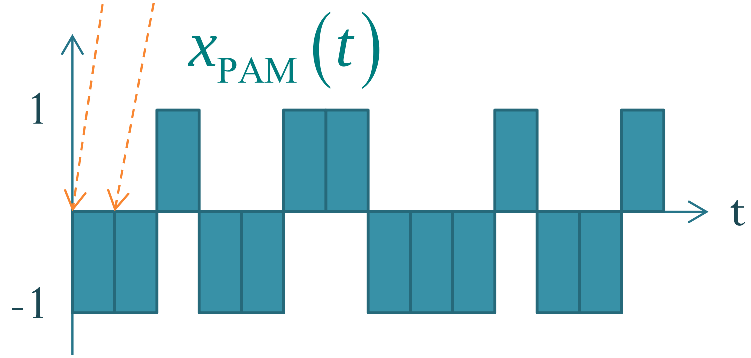
$$g(t)\cos(2\pi f_c t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2}G(f-f_c)+\frac{1}{2}G(f+f_c)$$

$X_{\mathrm{PAM}}(f)$ (3/4)





m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

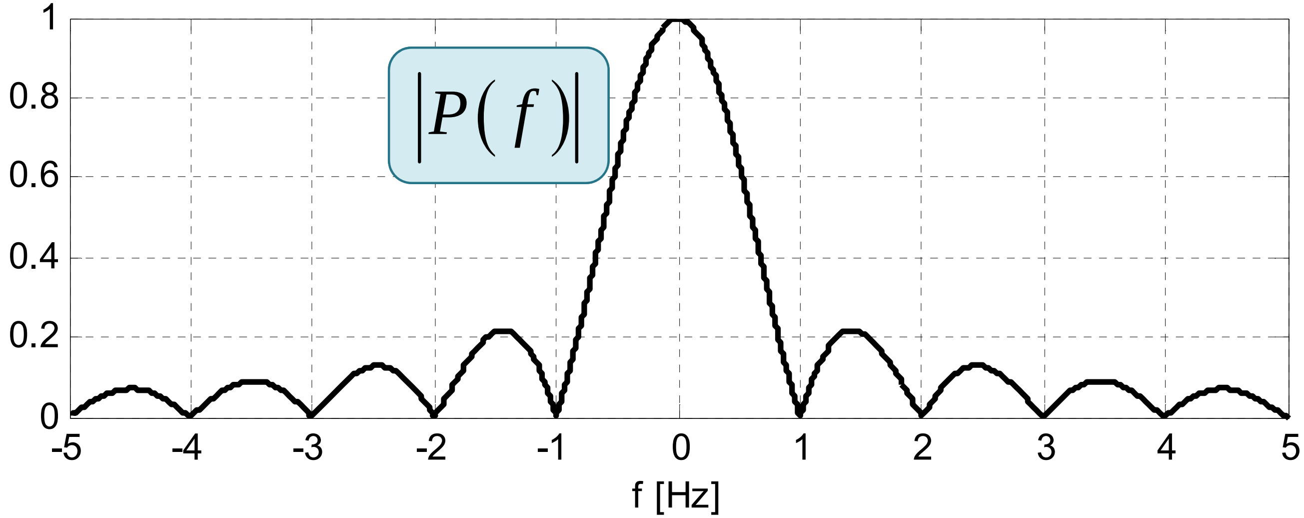


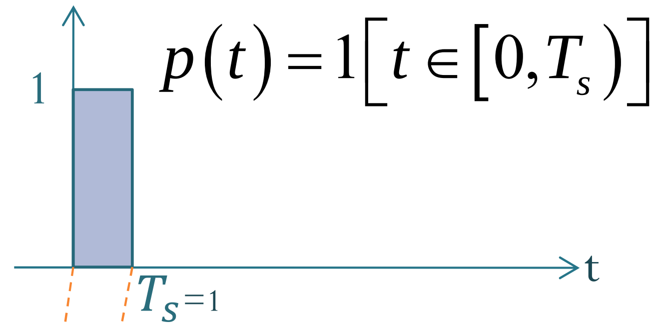
$$x_{\mathrm{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

$$\xrightarrow{\mathcal{F}} X_{\mathrm{PAM}}(f) = \sum_n m[n]\, P(f)\, e^{-j2\pi f n T_s}$$

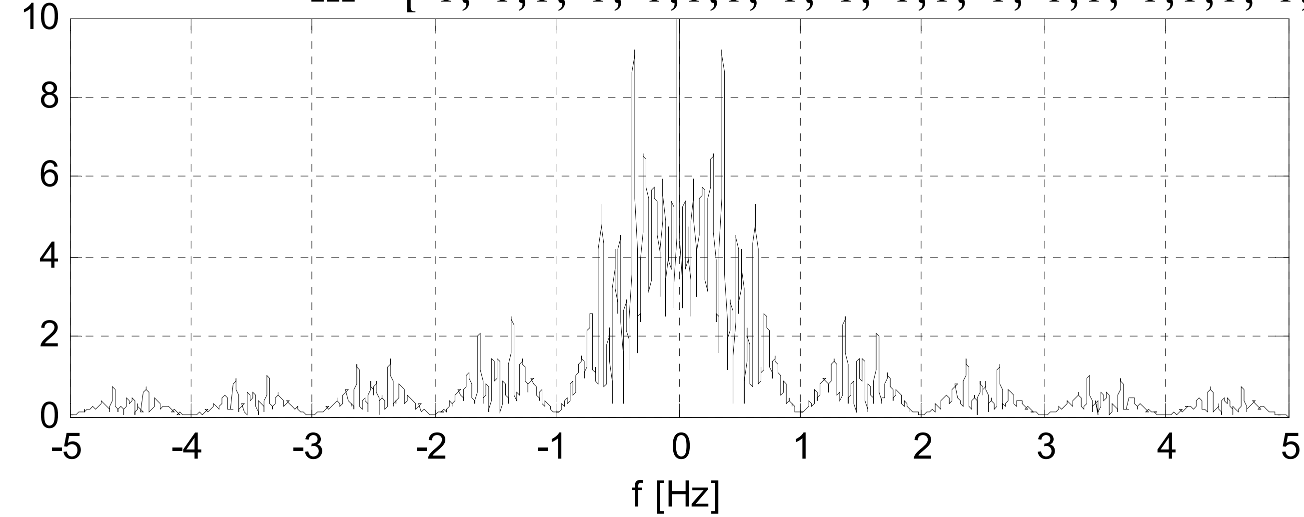
$$= P(f) \sum_n m[n]\, e^{-j2\pi f n T_s}$$

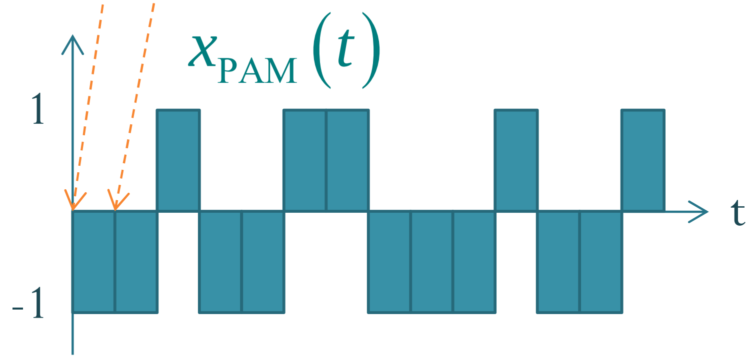
$X_{\text{PAM}}(f)$ (4/4)





m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

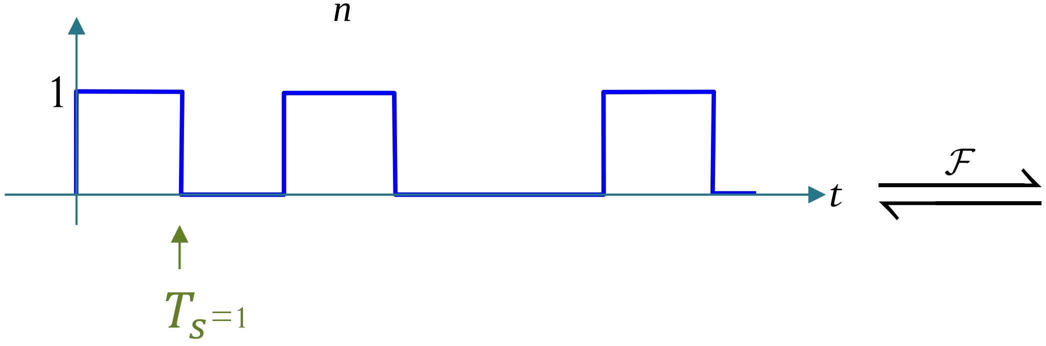


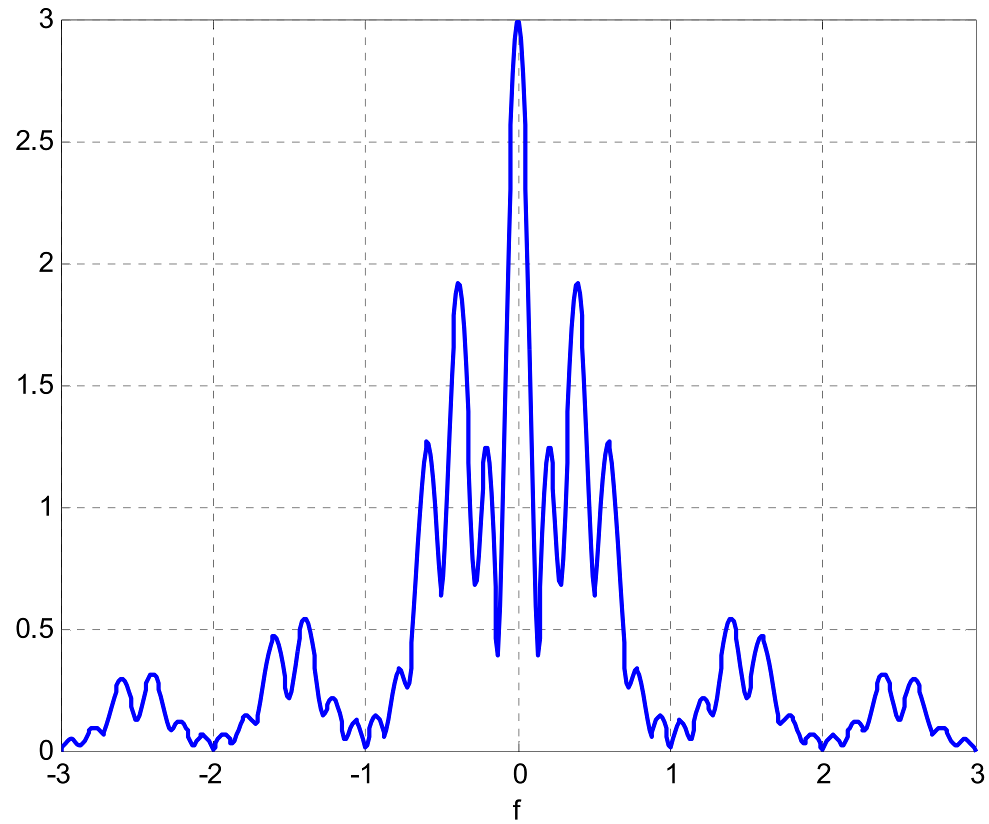


$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s) \xrightarrow{\mathcal{F}} X_{\text{PAM}}(f) = P(f) \sum_n m[n]\, e^{-j2\pi f n T_s}$$

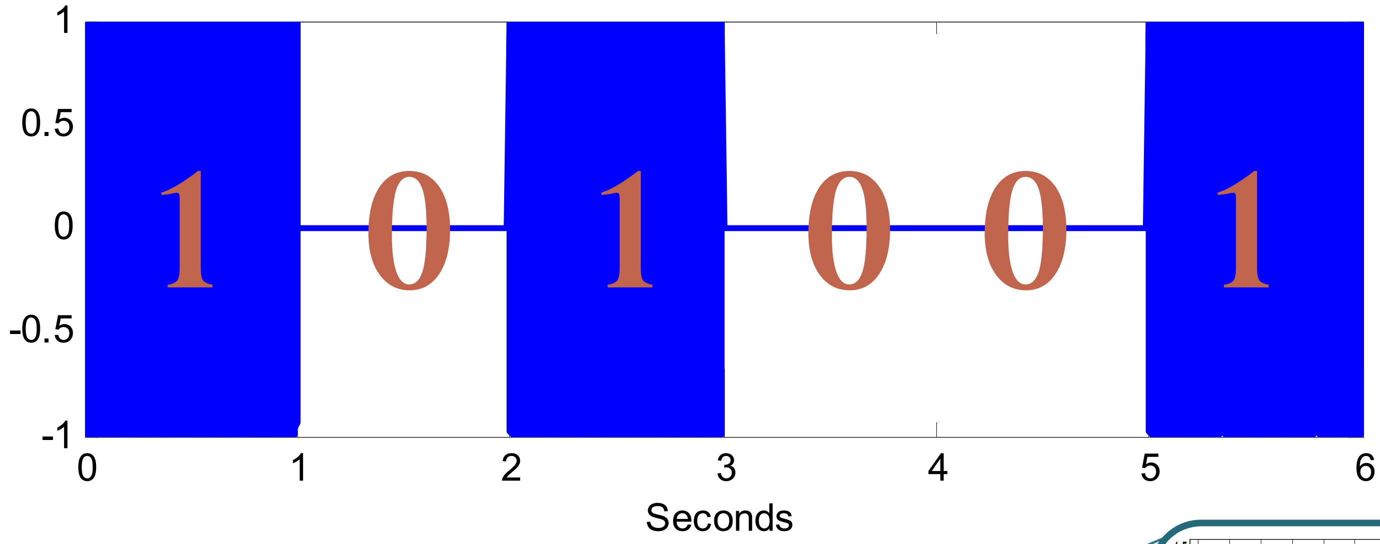
A revisit to an earlier OOK Example

$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$





Spectrum of ON-OFF Keying



$f_c = 100$ Hz
Bit rate = 1 bps